Economic Thought 9.1: 72-95, 2020

Mathematical Analysis as a Source of Mainstream Economic Ideology[1]

Vlassis Missos, Center of Planning and Economic Research (KEPE), Greece
vmissos@kepe.gr

Abstract

The paper contends that neoclassical ideology stems, to a great extent, from mathematical analysis. It is suggested that mainstream economic thought can be comprehensively revisited if both histories of mathematical and economic thought are to be taken collaboratively into account. Ideology is understood as a 'social construction of reality' that prevents us from evaluating our own standpoint, and impedes us from realising our value judgments as well as our theories of society and nature. However, the mid-19th century's intellectual controversies about the validity of mathematical thought, truth and knowledge can procure new interesting insights concerning the ideological stance of the first marginalists. In this respect, the methodological categories of analysis and synthesis serve as the basis for the crucial distinction between *old* geometry and mathematical analysis, indicating that the discipline of mathematics has its own history of fundamentally unresolved disputes. Lastly, this may also shed some light on Alfred Marshall's peculiarly reluctant attitude towards the use of mathematical analysis in his work.

Keywords: mathematical analysis, synthesis, neoclassical theory, Marshall

1. Introduction

The evolution of mathematical ideas, similarly to economic ones, adheres to a rather impressive and thought-provoking history that could rub our old dusty soil with fresh water. The present paper aims to show that economic practitioners, after recognising that

> 'the mechanical way in which [differential] calculus sometimes is taught fails to present the subject as the outcome of a dramatic intellectual struggle which has lasted twenty-five hundred years […] and which will continue as long as man strives to understand himself as well as nature' (Courant, foreword in Boyer, 1949, p. i)

might feel the need to revisit aspects of their own established knowledge. In accordance, the paper intends to contribute toward the research lines that have already been set forth by Weintraub (2002), whose pivotal work concerning the relation between economics and mathematics take the histories of thought of both disciplines into consideration. What is more, the important points raised by Dow (2003) are also highly valued, especially the one referring to 'the history of the development of mathematics and its relations with economics' (Dow, 2003, p. 547). Looking into this stimulating field proves refreshing and renders new interesting

[1] This research is co-financed by Greece and the European Union (European Social Fund-ESF) through the Operational Programme 'Human Resources Development, Education and Lifelong Learning' in the context of the project 'Reinforcement of Postdoctoral Researchers-2nd Cycle' (MIS-5033021), implemented by the State Scholarships Foundation (IKY).

insights of the mid-19th century theoretical developments, which have culminated in the writings of highly esteemed economists such as Cournot, Gossen, the first marginalists and Marshall. Mainstream economic ideology can be fairly revaluated through the ontological controversy that took place within the branch of natural philosophy, a blend of what is today broadly known as classical physics *and* mathematics.

The argument below emanates from the idea that the history of Political Economy should begin to incorporate and, if necessary, revise several aspects of the historical development of mathematical thought into its own curriculum, so that the obscured ideological views underlying the various conceptions, can come to the fore. Investigating the social content of mathematical ideas would provide evidence that a clear cut dichotomy between natural and social sciences is not that definite – to the degree, at least, that it is currently believed to be.[2] Nature does not speak for itself, neither does its heart open to us widely to reveal the laws under which it is governed. The modes of reasoning that we use towards understanding nature, encompass parallels and coherent views of society. In this respect, ideology, the imposed array of prejudices which limits the scope of any research, no matter how thoroughly undertaken, veils the spectrum of our vision, blinds our judgment or, under particular circumstances, turns into a deeper understanding of phenomena and reflects upon the 'impartiality' of our point of view. However, if one 'seeks to understand and allow for these prejudices (and he is certainly morally obliged to do so) then he must examine his whole individual and social history and background' (Macfie, 1963, p. 225). That being so, careful and laborious contemplation requires considerable effort in tracing the roots of the so-called 'historical puzzle' with regard to the 'attachment of almost every economist […] to the mechanistic dogma' (Georgescu-Roegen, 1971, p. 2).

As already mentioned, this standpoint is not new to our field. Reputable scholars have undertaken the painstaking task of bringing together the histories of these two apparently distinct areas. Among the most prominent studies, the comprehensive work of Phillip Mirowski (1989) has attracted a lot of attention due to its unique manner of inspecting the subject. Mirowski maintains that the dominion of neoclassical economics is, to a large extent, owed to the fact that it arose as an immediate fruit of mathematical physics' 1870s rigorousness. His historical method of research is carried out by tracing the gradual application of the mechanistic techniques into this branch of Political Economy and by encountering the analogies between the several concepts of the two fields, such as that of 'utility' and that of 'energy'. In another fashion, Weintraub (1991) examines the history of static and dynamic versions of equilibrium theories following the 1930s, focusing on the multiplicity of ways economic scholars have interpreted it. In that study, equilibrium is captured as a central feature of economic theorising that lacks empirical justification, while being part of an arcane language used among the field's associates. This neo-platonic[3] or constructivist view of economic knowledge has been criticised on the grounds that it fails to recognise the 'definite assertions about the nature of reality' (Blaug, 2003, p. 152) it involves and the political statements it implies.

[2] Theocarakis (2014) has traced the origins of this distinction to be at least as old as the *Dialogue* of Galileo Galilei (1632). 'If what we are discussing were a point of law or of the humanities, in which neither true nor false exists, one might trust in subtlety of mind and readiness of tongue and in the greater experience of the writers, and expect him who excelled in those things to make his reasoning most plausible […] But in the natural sciences whose conclusions are true and necessary and have nothing to do with human will, one must take care not to place oneself in the defense of error' (Galileo Galilei, 1632, pp. 53-54).

[3] The neo-platonic approach considers mathematical ideas as having no necessary foothold within our surrounding physical universe. 'Mathematical reasoning was seen as a mental art rather than a physical one with […] empirical unverifiability of a theoretical process' (Whiteside, 1961, p. 185). By the late 18th and early 19th century however, this view had been abandoned. The Scottish mathematician John Leslie was contented that 'Geometry is […] founded on external observation' (Leslie, 1811, p. 1).

The primary reason why both histories of thought require a combined effort in further examination is related to the paper's main aim, showing that mainstream economic thought has been ideologically influenced by a specific branch of mathematics – that of *analysis*. Be that as it may, a survey on ideology falls out of scope of this paper. Ideology is here treated in a narrow sense and as a meaningful term to help us bring forward the intense relation between mathematical analysis and neoclassical thought. In that respect, the manner in which 'social constructions of reality' (Berger and Luckmann, 1966) are being built is not investigated below. For our purposes, it would suffice to add that ideology is understood as an indispensable part of the theories themselves, with regard to both society and nature, and not just as an exclusive relation between the observer and the observed. It is broadly approached as a feature that characterises the totality of human thought, both scientific and moral.

Ideologies can be conceived as pre-constructed frameworks found upon the sets of beliefs, intending to infuse meaning and values to the latent material mediating social conduct. The representations they offer to natural and social life are themselves partial and to that respect, Heilbroner (1990, p. 103) classifies them under the field of rhetorics i.e. the art of persuasion. Some aspects of the social or natural phenomena are concealed whereas others are magnified. Through this angle, the critique posed by Meek (1967) suggesting that no scientific approach in the field of economics can exempt itself from 'ideological distortions' is in accordance with the main argument below. By 'distortions', Meek does not contend that ideological views deviate from an objective or absolute knowledge of the truth. Distortions refer to frameworks of perception used by economists, emphasising on specific aspects while others are left aside. The mixture of ideology and science, Meek suggests, cannot be sorted out. In what follows below, this thesis is extended in order to maintain that mathematical thought is of ideological essence and its vision of the world has, moreover, influenced the economic perception of society.

In section 2, the ideological aspects of mathematical thought are discussed in relation to Political Economy. Section 3 refers to a rather short part of an extensive debate between the main figures of the Victorian era of natural philosophy, illustrating that the history of mathematical thought is better understood when approached as a controversy of beliefs instead of as an exegesis of proving the truth. It is claimed that algebra does indeed constitute a mode of reasoning and not just a 'perfect system of language, adapted to the notions and relations that we need to express' (Jevons, 1871, p. 5). Objections are not raised against symbols or symbolic treatment, but against the ideas incorporated and the political and social extensions to which they refer. Section 4 traces the seminal idea of analytical thought, while in Section 5, some remarks from the main representatives of the dominant paradigm relating to the subject of mathematical analysis and economics are discussed. In that context, Alfred Marshall is approached as a special case requiring careful treatment.

2. A Politico-mathematical Economy

Recent contributions have examined several aspects of the relation between mathematics and economics. Among them, Milonakis (2017) can be singled out for his emphasis on the role of social and political history in the development of mathematical formalisation of economic ideas. His argument unfolds in the line that classical political economists were more cautious, even reluctant, when expressing their theoretical statements mathematically since, to them, the social and historical qualities that perish when mathematical language is applied, were important elements of their arguments. From a different perspective, Martins (2012) has claimed that mainstream economic theory incorporates a specific trend of mathematical thought, which draws heavily on the Cartesian, algebraic tradition. He goes on to assert that economists who did care about the realistic conceptualisation of their theories – such as Marshall, Keynes and Sraffa – held a different stance concerning the mathematisation of economics, while their techniques bore close resemblance to that of Newton. What is not pointed out in neither of these approaches, however, is the evaluation of the entrenched ideology, existing in all principles of human thought, mathematics be no exception.

Studies considering the issue of ideology in economic thought, usually place emphasis to Schumpeter's *History of Economic Analysis* (Heilbroner, 1990; Dobb, 1972; Meek, 1967; 1957). Considering our purposes, the need for resorting to the *History* is two-fold. To start with, Schumpeter regards ideology as a qualitative property that can easily be detached from our theoretical insight and secondly, he keeps a strong position in favor of analysis as a practical and unassailable tool of scientific inquiry. '[M]athematical economists' he says, 'form no school in any meaningful sense of the term, any more than do those economists who read Italian' (Schumpeter, 1954, p. 922). Consequently, for Schumpeter, the main instrument of economic thought is held to be unbiased, having no effect on the manner in which the concepts of our discipline are constructed in our minds and theories. Mathematical thought is interpreted as a solid and unified field of uncontroversial character and knowledge, which exclusively corresponds to its analytical trend that, at least in Britain, dominated the subject around the mid-19th century. It would, thus, come to no surprise that in his *History*, Schumpeter concentrates on the 'history of those techniques' which he characterises as 'scientifically admissible'. To succeed in this, he ensures to advise that 'a number of obstacles will have to be removed before we can feel sure of our ground – the most serious one carrying the label *Ideology*' (Schumpeter, 1954, p.6, my emphasis). What still remains in darkness though, is that the very preference preceding the application of analytical techniques in which the ideological features of our vision of economic issues are deemed to be removed, contains a lot of ideology *in* and *of* itself.

The term 'mathematical economist' used by Schumpeter, has also been endorsed by Samuelson as he who 'need not be of any political persuasion' (Samuelson, 1960, p.25). Here, obviously, Samuelson attempts to detach mathematics from its rhetoric or, to recall Heilbroner, its ideological aspects. Others, have sought for the antecedents of mathematical economists even before the age of Cournot (Robertson, 1951), as if they are non-historical creatures that have always existed, capable of applying the techniques of differential calculus in their work, while, Theocharis (1993), saw the application of mathematical analysis in economic theory as the gradual progress of accumulated knowledge.

Discussion with respect to the relation between ideology and 'mathematical economics', however, has produced a series of counter arguments and competing approaches as that between Lawson (2012) and O'Boyle and McDonough (2017). The former's claim that 'mathematical methods and techniques are essentially tools' (Lawson, 2012, p. 3) deviates substantially from our current view, in which mathematical ideas are recognised as social products fusing life and essence in the rudimentary forms of expression. Next, Lawson, takes up Kline (1964) to stress out that the modern tendency towards formalisation emanates from a well-rooted, culturally wide-spread aesthetic prejudice, which grants mathematical expression with an air of sophistication. The narrative takes no account of the evolution of mathematical ideas in themselves neither of the differentiated views of the world their several trends can offer. William S. Jevons, for example, discerns between a 'true' and a 'false mathematical theory', applying the first one so as to correct the theoretical impasses in which Smith and Ricardo had fallen after following the latter (Jevons, 1862, p.487). As it may be the case, a distinction between mathematical theories has also been acknowledged by O'Boyle and McDonough (2017) when endorsing the concept of 'mainstream mathematics'. One of their main arguments is built on the idea that 'marginalist mathematics were entirely subservient to an ideological utilitarianism' (O'Boyle and McDonough, 2017, p.26). The authors suggest that the symbolic reasoning that stems of particular social context is value-laden, attempting to reproduce dominant thinking and social relations. As it is indicated below, a particular strand of mathematical thought has infused its own ideology into mainstream economic theory.

Within the mainstream approach, the Political Economy of Adam Smith is commonly understood as gravitating towards the idea of an invisible hand (WN, IV.ii.9). This popular metaphor blends together in a mixture of mystical and logical forces, operating as a compromise that mediates between the counter aims of antagonistic social groups, precipitating the realisation of unintended outcomes (Aydinonat, 2008). Neoclassical economics, in the broad sense of the term, has given significant weight in this allegory and treats it as a short-cut argument in favor of laissez-faire policies bestowed by Smith's authoritative status. Throughout neoclassical analysis, the virtuous ends of individual actions remain secret to them and are revealed only after having been engaged with the attainment of their own private gain. However, opposing social and individual aims are brought forward in a manner consistent with the thought customs of the entire scientific epoch of 18th century, when market outcomes were still questioned on the grounds of their moral qualities. Skinner (2012, p. 163), for example, maintains that Smith was influenced by the pioneering concept of François Quesnay's tableau, in which the economy was contemplated as a system resembling a human body, in which blood flows autonomously through its veins. In this primitive model, 'blood' flows circularly and independently of our personal will, bearing a central implication on the fact that as we cannot claim any authority or subjective judgment upon the proper and healthy function of the body, economy should similarly be given the freedom to operate according to patterns and ideals of natural liberty. Human heart keeps beating irrespective of man's conscious desire to do so. Interestingly though, neoclassical theory, Dobb explains, incorporated an ideal moral form of natural liberty as it was derived from the application of differential calculus techniques.

> '[T]he term 'Neo-classicists' is not entirely inappropriate; for what the Cambridge school has done is to divest Classical Political Economy of its more obvious crudities, to sever its connection with the philosophy of natural law, and to restate it in terms of the differential calculus' (Dobb, 1924, p. 68).

August Cournot, the pioneer in applying the method of calculus in Political Economy, had contributed in a rather sophisticated way to give the impression that his mathematical analysis was an objective and unprejudiced judge of knowledge.

> 'I have not set out to make a complete and dogmatic treatise on Political Economy; I have put aside questions to which mathematical analysis cannot apply' (Cournot, 1838, p. 5)

Dogmatism was recognised as believing in anything but mathematical analysis, which systematically was becoming the tool of separating the wheat from the chaff! The moral status of this technique was spread and became one of the main tools for identifying appropriate and inappropriate interpretations in the years of Alfred Marshall's youth. Marshall seems to be familiar with the practice of using that kind of mathematical prejudgment as the touchstone for economic truth[4], when, in a letter to J. B. Clark, he writes:

[4] Jevons (1862, p. 488) understands this technique as 'the touchstone which can decide between truth and error'.

> 'My acquaintance with economics commenced with reading Mill, while I was still earning my living by teaching Mathematics at Cambridge; and translating his doctrines into differential equations as far as they would go; and as a rule, rejecting those which would not go' (in Pigou, 1966, p. 412).

However, as suggested in the citation, during his mature years, Marshall, an astute mathematician, did not share the same enthusiasm with Jevons on the implementation of differential calculus in economic theory. The latter was sought for a 'true theory of Economics' adhering to the 'appropriate branch of mathematical science […which] consists in applying the differential calculus…' (Jevons, 1871, p. 3), whereas the former contends that 'the argument in the text is never dependent on them […]. The chief use of mathematics in economic questions seems to be in helping a person to write down quickly, shortly and exactly, some of his thoughts for his own use' (Marshall, 1961, p. xiii). On that account, among the main figures of neoclassical tradition, the views concerning the validity or usefulness of mathematical analysis differ substantially.

Similarly to Jevons, Walras wholeheartedly believed in the practical as well as ethical importance of this 'science of small increments.' His groundbreaking research was concerned with the question of 'how could political economy be simultaneously a natural science and a moral science?' (Walras, 1874, p. 56). In its pure form, Political Economy should blend the 'blind and ineluctable forces of nature' along with 'those which result from the exercise of human will […] free and cognitive' (Walras, 1874, p. 61). Such a positivistic and ethical project would require a scientific field 'as an explanation of what is and as a programme of what ought to be' (Walras, 1874, p. 60). Moreover, the latter would correspond to an idea of justice secured under the process of free exchange in the market. It was considered legitimate for Political Economy to use the tools applied in natural science, even if its objects of inquiry could not be exactly measured. With reference to these tools, whenever barter takes place in the market

> 'there are none by which to measure the intensities of needs of traders. But it is of no importance, since each trader takes upon himself this operation, consciously or unconsciously, and decides for himself if these last needs satisfied are proportional to the values of the commodities. Measure, that is, the comparison of quantities and quantitative relations, is not prevented by its exterior or interior quality, according to whether the measurable facts are physical or psychic. Consequently, neither is science denied its mathematical character' (Walras, 1909, p. 213).

Mathematical analysis (and the technique of utility maximisation) was regarded as having the ability to abstract and squeeze out the constitutive juice of a particularly meaningful and complex occasion that could distract the observer's attention. The fairness of the market outcome was thought to be realised through the use of this tool, independently of human will. '[T]his price of wheat does not result either from the will of the buyer or from the will of the seller or from any agreement between the two' (Walras, 1874, p. 69). The magic of the marketplace was up to a point reasonably analysed. How, thus, to justify further the occult forces upon which this outcome was produced? The remaining unexplained part of the final compromise between the opposed individual interests was charged on Smith's invisible hand that, unfortunately, was all that would be left hitherto from his theory to remember.

The marginal school did not arise as an immediate outcome of scientific machinery, but rather as a gradual historical process. A great number of studies, so far, have emphasised the fruitless projects aiming to transform the body of knowledge of economic theories into a clear-cut positive one, the concepts of which will be directly produced and best interpreted by the toolbox of mathematical analysis. A step backwards, however, may provide us with some vital space to consider that knowledge, whether of natural things or of concepts of pure reason, is neither straightforward nor certain. On the contrary, one may argue about the ethical ground on which it has been instituted. For instance, Werner Stark's (1943) *The ideal foundations of economic thought* can be viewed as an attempt to revisit the knowledge of our discipline through the old-fashioned lenses of its ethical origins. It has been pointed out that Stark appears to be 'hostile to the modern tendency to analyse society in terms familiar to those employed in the realm of Nature. He is hostile alike to capitalism and the emphasis of most contemporary economic thought' (Dobb, 1944, p. 153). This observation is crucial, for it seems to acknowledge the presence of an imperceptible thread linking epistemological and political views. Indeed, the former idea has certainly shaped the research method of the author himself whose investigation of logical and metaphysical problems set forward by Leibniz, led him to suggest that

> 'behind the formal problem of the relation between the continuum and its points, between whole and parts, lies hidden the material problem of the relation between the universe and its elements, between society and individuals. His sober mathematics contains a deep cosmology and sociology' (Stark, 1943, p. 27).

Stark clearly suggests that one of the major mathematical problems Leibniz was confronted with, wasn't thought out as an issue of mere scientific speculation falling solely within the realm of enlightened reason. In a quite different perspective, the multiplicity of its purpose extended far beyond the intellectual contentment of providing rigorous answers to questions posed by logic, just 'for the glory of human thought' (Moret, 1915, p. 6, in Ingrao and Israel, 1990). As was the case, mathematical reasoning was exercised like an *a priori* instrument of supreme ethical qualities and unquestionable benevolence, able to produce outcomes that were consistent with definite political arguments and, at the same time, reasonably proven. The divine merit of mathematical thought[5] was portrayed in the belief that it served as a 'ink between ourselves and the best and noblest benefactors of our species' (Herschel, 1830, p. 16). As a precondition, in case a mathematical solution was offered to a problem of any kind, whether of political, social or natural essence, that solution and every logical extension with which it was associated had to, consequently, be of undoubted ethical status. Hence, the mathematical ideal of a society without tensions, class struggles and social disputes, was envisaged as a concomitant result of accepting the validity of that mode of reasoning.

Mathematical knowledge was credited as having a noble influence on the mind, as a way of liberating our thoughts from temper, curbing and limiting the sharp, annoying edges of a passionate character. Widening the range and disseminating its ideas among the competent individual members or groups of society, as it was seen from the writers of classical Political Economy, would produce, as a result, calm and peaceful relations between them and would promote barter at a higher level, where reciprocity (Theocarakis, 2008) would be the outcome of factors other than the cunningness of self-interest.

[5] In his *Theodicée*, Leibniz, supports that human rationality is analogous to God's divinity. 'By the new infinitesimal analysis the geometers do only somehow imitate God' (in Stark, 1943, p. 34). For a discussion concerning the relation between political economy and theology in 19th century Britain see Oslington (2017). Cannon (1978) also points out that, for the period under consideration, 'Mathematics is a sign of truth; it is one of God's languages' (Cannon, 1978, p. 267). Lastly, a more general approach of the link between economics and religion is offered in Dow (2005).

> '[T]he calm and dispassionate interest with which they [sciences] fill the mind renders them a most delightful retreat from the agitations and dissensions of the world, and from the conflict of passions, prejudices, and interests in which the man of business finds himself involved' (Herschel, 1830, p. 16).

The image of a peaceful society characterised by a harmony of interests among its parts ought to be approached as a deeply rooted ideological element of mathematical justification. The fact that such a society is not present or realised did not discourage John W. Hershel from illustrating the necessary logical requirements of its existence. Thus, in more or less the same spirit as Leibniz, Herschel revisits the issue of blending and synthesising the competed interests between individuals and society, alluding to the political extensions of accepting the assumptions needed for the paradigm to operate within reason. The discussion of the mid-19th century regarding the interpretation of mathematical ideas had certainly focused on the relations between natural objects, but its results were also mirrored upon society and extended upon an ideal state of behavior.

> 'There is something in the contemplation of general laws which powerfully induces and persuades us to merge individual feeling, and to commit ourselves unreservedly to their disposal; while the observation of the calm, energetic regularity of nature, the immense scale of her operations and the certainty with which her ends are attained, tends, irresistibly, to tranquillise and reassume the mind, and render it less accessible to repining, selfish, and turbulent emotions' (Herschel, 1830, p. 16).

Herschel addresses here two very important points, as far as the general mathematical view of society is concerned. The first and less straightforward corresponds to the occult quality our contemplation of natural order is affiliated with. It would have been plausible to infer that the influential characteristic feature – with which the reflection on general laws is associated – on individual behavior is, on the one hand, certified but on the other, pursued. The above passage implies that the noble and valuable consequences of mathematical reasoning act upon the individual's character in such a manner, causing all distinct subjective opinions to be self-willingly mitigated under the unifying effect of natural and undeniable truth. Secondly, in psychological terms, individuality is supposed to disengage from all evil qualities (selfishness, turbulent emotions) that do not permit the unification of interests under a common purpose. Therefore, pure scientific knowledge could frame the issues of political economy and would concentrate upon an ideal image of society, illustrating a state of mind which, though not historically present, is openly claimed and theoretically attained. According to this interpretation, general laws, whether confirmed by evidence of experience or not, would have been imposed by necessity as a form of ideology.

3. A *right* mathematical theory for economics

Elemental aspects of ideology in mathematical thought can be traced within the mid-19th century debates in Britain, concerning the nature of its knowledge. Joan L. Richards' (1988) *Mathematical visions: the pursuit of geometry in Victorian England* stands out as one of the most thorough treatises with reference to the major controversies on the different mathematical perspectives, which was carried out among contemporary natural philosophers, at a time when several important scientific advances – such as non-Euclidean geometry – were about to step up, gaining broad and rapid recognition. The author offers a historical account of the open-ended intellectual debates concerning the ontological basis of mathematical knowledge, pondering its effects on education. By the 1870s, she concludes, the Euclidean foundations of geometry had been vividly questioned, while geometrical conceptions were progressively interpreted in a formal rather than a descriptive way. Amongst these two broadly conflicting views, numerous other approaches can be identified as byproducts of an intense intellectual interaction among scholars of science. Such a decisive shift on mathematical vision was able to convert the general understanding of natural as well as social life. In a meaningful way, this controversy is indicative of the ideological ground upon which the broad range of human knowledge has been erected and it provides new unconventional ways for deepening our understanding of its relation with the epistemological aspects hidden behind the historical process of gradual transition from political economy to economics (Milonakis and Fine, 2009, ch. 12; Halevi et al., 2011).

The dispute between mid-19th century mathematicians in England was primarily focused on the manner in which true human knowledge is attained. It was taken for granted that truth was itself – as God was – single and undivided and that geometrical reasoning could reveal aspects of its unifying singular nature. Science was considered as the norm towards it (Cannon, 1978, p. 271). During such a process, a significant part of this intellectual debate was framed by two incompatible epistemological approaches. The first was the nativist, which roughly asserted that true knowledge of natural realities was somehow innate to them, whereas reason served as the guiding light towards it. The belief in the intrinsic qualities of the matter can also be found in some of Thomas De Quincy's (1844) early contemplations on utility as a property inherent to commodities. The second was the empirical approach, which argued in favor of experience as a means of verifying knowledge through senses. William Whewell, in his work *The philosophy of inductive sciences* (1837) supports the nativist view, elaborating upon the fundamental conception of necessary truths. Whewell resorted to statements of undisputable certainty in order to form a solid basis for human knowledge, from which all other subjects could stem. He consequently maintained that, owing to its occasional character, experience could not procure a safe road towards the ideal qualities of truth, and to furthermore generate the required level of universality.

> 'Necessary truths must be universal truths. If any property belongs to a right-angled triangle necessarily, it must belong to all […] necessity and universality cannot possibly be the mere results of experience […]. That from experience we obtain much knowledge which is highly important, and which could not be procured from any other source, is abundantly clear […]. Experience must always consist of a limited number of observations […] being thus unable to prove a fact to be universal, is […] still more incapable of proving a truth to be necessary. Experience cannot indeed offer the smallest ground for the necessity of a proposition. She can observe and record what has happened; but she cannot find, in any case, or in any accumulation of cases, any reason for what must happen. She may objects side by side; but she cannot see a reason why they must ever be side by side […]. She contemplates external objects; but she cannot detect any internal bond, which indissolubly connects the future with the past, the possible with the real. To learn a proposition by experience, and to see it to be necessarily true, are two altogether different processes of thought' (Whewell, 1837, pp. 61-64).

As it becomes apparent, the idea that truth must hold under any circumstances provides sufficient reasons for the property of universality to be established. Experience, on the other hand, is considered to be unqualified to testify for truth's necessity on the grounds that human senses are bound by 'a limited number of observations' lacking, thus, the appropriate legitimacy to generate catholic statements. In the previous passage, however, as can be observed, the efficacy of experience is denounced, since it "cannot see a reason" able to connect the cases procured by observation. Necessary truths acquire this property beforehand, as a precondition, but it is this very act of selecting one instead of another process of thought that makes room for ideological considerations. What if choosing this approach, instead of the other, could offer some potential for better results? What is the criterion for judging it? The prospect of this answer may lead us to moral as well as political arguments.

Whewell's use of reason is somehow forced, as if a previously imposed order of things had been authoritatively dictated. Something that stands beyond our range of influence compels us to allude to a particular succession of things. Causes and effects, Whewell demonstrates, are recognised through senses, but it is the very idea of *causality* that has its origin in the mind itself, which has already been formed as a category that prompts us to pursue for it. Thus, our observation is ultimately preconceived. 'But that every event has some cause, experience cannot prove any more than she can disprove. She can add nothing to the evidence of truth, however often she may exemplify it' (Whewell, 1837, p. 167). The metaphysical bond between observations could not be derived directly through senses but was, instead, necessitated as a crucial mystical element which was denied by the empirical approach. John W. Herschel stands as a representative of the opposing belief, that true knowledge can spring from experience and can be mediated through science.

> 'The truths it [science] is conversant with are necessary ones, and exist independent of cause. There may be no such real thing as a right-lined triangle in space; but the moment we conceive one in our minds, we cannot refuse to admit the sum of its three angles to be equal to two right angles […] No one causes or makes all the diameters of an ellipse to be bisected in its centre […]. But in natural science cause and effect are the ultimate relations we contemplate; and laws, whether imposed or maintained, which, for aught we can perceive, might have been other than they are […]. We have thus pointed out to us, as the great and indeed only ultimate source of our knowledge of nature and its laws, EXPERIENCE; by which we mean not the experience of one man only, or of one generation, but the accumulated experience of all mankind in all ages […]. Experience once recognised as the fountain of all knowledge of nature, it follows that, in the study of nature and its laws, we ought at once to make up our minds to dismiss as idle prejudices […] any preconceived notion of what might or what ought to be the order of nature in any proposed case, and content ourselves with observing, as a plain matter of fact, what is.' (Hershel, 1830, pp. 75-78)

What is actually claimed, is that the true meaning of scientific ideas stems autonomously or even, naturally, from its own reasonable properties. According to Herschel, no one causes its appearance or controls its exposure at will and, at the same time, no subjective opinion intervenes. The Victorian scholar held that science fulfills this noble purpose, operating like a vehicle which leads us safely to truths of unassailable validity. As a consequence, knowledge as such, is ready to be perceived through immediate laws or norms able to be observed by those who have accepted to wear the appropriate glasses for seeing it. Contemplation concerning a potentially alternative order or state of things is forbidden and perceived as subjective prejudice. The observer is obliged to remain impartial, starting with gazing serenely at the image of a lawful natural state and the aim of contemplation is to examine these laws under the belief that they represent the truth and not a predefined belief.

These two theories, apart from their epistemological differences, reveal certain distinct attitudes of scientists themselves. In the first one, knowledge is supposed to be out there waiting to be discovered, whereas in the latter, the very meaning is procured by the individual mind in which the appropriate method has been cultivated. Of course, in both cases, no one is free to express prejudgment while he is strongly obliged to accept that his own theory of approaching knowledge is the right one. A commonly accepted yardstick for discerning between falsity and truth was thus gradually formed and more than that, was also about to be applied to the field of Political Economy.

> 'I maintain that is only by going back and reconsidering the primary notions of the science that we can arrive at a true theory of economy, and be enable to distinguish between the true and the false […] The laws of political economy must be mathematical for the most part, because they deal with quantities and the relations of quantities' (Jevons, 1862, pp. 479-480).

Here, Jevons seems to offer his own answer to a historical argument posed by Scottish mathematician Thomas Reid in his *An Essay on Quantity*, in which he denies the application of mathematical reasoning to subjects relating to moral ideas of virtue and merit. "Pain and pleasure admit various degrees, but who can pretend to measure them?" (Reid, 1748, p. 506). The enthusiastic spirit of Jevons, on the other hand, also made him to acknowledge that unless an economist is equipped with the appropriate mathematical theory only then can he study this science of quantities in an upright manner.

> 'Thus the chief difference between the old and the new doctrines is, that the old ones involve a crude and partially false mathematical theory and the new ones, as I venture to maintain, a true mathematical theory. This difference arises I believe, from overlooking the importance of a thorough analysis of the notion of utility' (Jevons, 1862, p. 480).

One, however, may wonder, what were 'the old and the new doctrines' to which Jevons relies? In Britain, the analytical method of inquiry, implied in the aforementioned part, was introduced with a relative delay. Olson (1971) has illustrated that the retarded arrival of analytic mathematics and the complete substitution of ancient or "synthetic" geometrical thought were due to the prominent role the Scottish mathematicians attributed to Common Sense, i.e. to the well-rooted habit of offering sensory referents for all meaningful terms.

Besides the intriguing question of how mathematical truths were actually founded, the vast majority of the examples offered by scholars in Britain, 'were consistently geometrical as opposed to analytic' (Richards, 1988, p. 39). The descriptive approach which was almost unarguably followed in their writings, permitted for a lower degree of abstraction, alluding to concepts – like that of 'space,' for example – which corresponded more easily or even immediately to our senses. In the years to come, the reluctant reception or even disdain of symbolic algebra can also be found in the observations made by Benjamin Jowett when reviewing Alfred Marshall's early draft of the *Principles* (Whitaker, 1972). In his correspondence with Marshall, Jowett expresses some sort of irritation concerning the use of algebraic symbols in political economy.

> 'Will you be surprised at my attacking you about Symbols? […] I seem to see that various persons such as De Morgan and Boole, have tried to applied mathematics to subjects which did not admit of their use & have rather deluded themselves & others [...] Now I do not object to their application to Political Economy, provided they are not regarded as a new method of discovery, but only as a mode of expressing a few truths or facts which is convenient or natural to the few whose minds easily adopt such symbols. Political Economy is human & concrete & should always be set forth in the best literary form: the language of Symbols may be relegated to notes and appendices' (Whitaker 1996, letter n. 148).

The quoted part constitutes a synopsis of the assessment espoused by many academic circles in mid-19th-century England, while it also makes its way prophetically, since Marshall's *Principles* were eventually published in the manner dictated by the last sentence. Diagrams, as geometrical demonstrations, were put altogether in footnotes, while mathematical analysis and differential calculus were placed in a distinct Appendix at the end of the book. Obviously, the above decision is opposed to that of Jevons' who interpreted "the symbols of mathematical books [as no] different in nature from language" (Jevons, 1871, p. 5). The impact these developments had on the subject matter of political economy was striking. Traditional issues like those on value or distribution were about to be restated and reformed following different methods of inquiry, influencing the entire terrain of political argumentation.

Two questions effortlessly arise with regard to what the difference is between geometry and analysis. Should economists be concerned or able to distinguish between the two? Our answer to the second question is affirmative, adding that the changing views encompass crucial ideological aspects that fit to the development of the utilitarian ideals of marginalism.

4. Essential differences between synthetic and analytical perspectives

In this section we find it most appropriate to present various seminal ideas and elementary aspects of the great discourse about the relation between these two sister arts of mathematics. The controversy is very old and extensive.[6] However, a brief or introductory sketch of the essential features of old geometrical thought is considered important for the history of economic ideas. The theoretical inclination of neoclassicism towards mathematical analysis can be satisfactorily uncovered if the characteristic qualities of these categories (geometry and analysis) are appreciated.

The analytical method has a long history that goes back to ancient Greek philosophy and geometry (Hintikka and Remes, 1974). The word 'analysis' is complex, stemming from the Greek phrase *ἀνάπαλιν λύσις which means, 'reduction backwards'. According to a famous mathematical compiler of the antiquity, Pappus from Alexandria:*

> '[I]n analysis we assume what is sought as if it has been achieved, and look for the thing from which it follows, and again what come before that, until by regressing in this way we come upon some of the things that are already known'.

[6] For an exhaustive inquiry of the subject see Kline (1972).

Hence, we consider having already reached what we seek as being found and, afterwards, we attempt to proceed rationally until we arrive to a conclusion that is known beyond doubt. In other words, the truth of what we seek is *a priori* declared, but it would not be established until it is logically connected to a certain fact of knowledge. Conversely, analysis is followed by synthesis.

> '[B]y reversal, we assume what was obtained last in the analysis to have been achieved already, and, setting now in natural order, as precedents, what before were following, and fitting them to each other, we attain the end of the construction of what was sought' (in Henderson, 1993).

Exactness and rigorousness are being furnished through the reverse process of synthesis – *constructio* in Latin – as it was started from a known fact, building up a consequential reasoning towards what the starting point was in the process of analysis. With the latter, the product of analysis could be fully demonstrated, and this may explain synthesis' close intimacy with geometrical constructions using the ideas of line-segments, perfect circles as well as other figures. Simple geometrical demonstrations offered a descriptive sense to the proposed solutions that was essentially needed to serve as a basis for their existence outside the mind of practitioners and natural philosophers, filling the need for sensory evidence of the proposed solutions. In this way, solutions were being *demonstrated* not just logically proved.

For centuries, that old[7] way of geometrising was considered as one of the highest arts. With the publication of Descartes' *La Géométrie* (1637), however, the ancient analysis was rejuvenated and utilised in order to take a different step away from the descriptive qualities of human intelligence as a necessary requisite for validating its outcomes. By then, according to the main trend, analytical results were contemplated by the implementation of Euclidean geometry, that is, by the use of simple figures and line-segments, appearing as unmediated and direct affections in the mind. The need for constructing a geometrical solution was necessitated by the idea that its visibility constituted a basis for its existence. Descartes' novel canon, however, consisted a clear departure from this ancient-long tradition, for it was developed in order to treat the problems of geometry within a context of analytical and algebraic conceptions (Bos, 2001). Such uprooting was not a mere change of style but rather of essence reflecting the epistemological views on the nature of the attained knowledge. The groundbreaking canon maintained that geometrical problems could be transformed into equations of knowns (*a*, *b*, *c*…) and unknowns (*x*, *y*, *z*…) and then proceed by resolving it, i.e. analysing – *resolutio* in Latin – as if the problem had already been solved (Bos, 2001, p.303). Furthermore, Descartes asserts that the product of the line-segments *a* and *b* must be conceived in order for a new image of an $\overline{ab}$ line-segment, and not a framed area, to be represented in our minds. This would reduce the faculties of our senses for illustration. After all, affirmation through experience couldn't be justified completely as a geometrical rule, since there was no particular reasoning for the imposition of a necessary transformation of two line-segments into an area. This example is striking in showing how classical geometry metamorphosed itself into algebraic equations, with all the unknowns treated as if they were already known through the comprised relations with the givens – *data* in Latin – of the problem at hand. Once more, the analytical method was used as a means of abstracting the features that would allow for an interpretation of results through one's senses.

According to Redman (1997, p. 131), nineteenth-century scientific milieu envisaged the analytical method as a generalised application of chemistry in every other field of knowledge, replicating the process in which chemical compounds are resolved into its

[7] The term 'old geometry' is used to distinguish it from its later *analytical* counterpart.

elements. Resolution (analysis) begins with an equation or symbolic relational form, the validity of which is ultimately derived as a logical consequence of a hypothetical *a priori* accepted truth. A symbol is thought to be capable of representing and acquiring any potential meaning, either quantitative or qualitative. Thus, we ultimately end up with a complete abstraction or a formal equation, between the entities considered, in which the mathematical reasoning is inflicted as if being part of their nature. Analytical train of thought substitutes for all other qualities or features. In this respect, various historically specified terms of Political Economy were re-substantiated into a new form and theory that, whereas it looked like merely wearing a new garb, it was absolutely more than that.

On the other side, old geometry acquires a series of different characteristics. Firstly, it is closely tied to what our senses perceive as a three-dimensional space. Secondly, its diagrammatic solutions are demonstrated and arrested by our senses and, thirdly, it is mainly concerned with magnitudes and figures possessing a meaning of their own – no matter how simple these may be – also capable of being captured by our senses. Furthermore, an elegant demonstration of a geometrical solution may also satisfactorily respond to our need for aesthetic quality but moreover, all its terms and concepts are designated as sensory referents. Geometrical figures cannot be found in nature and, thus, should not be confused with that part of knowledge the validity of which is being found in experience. But, whether they constitute mere products of our mind or are able to be found as natural entities, they are observable and correspond immediately to our understanding as such.

Be that as it may, Isaac Newton's openly expressed distaste for analysis and his criticism against Cartesian symbolic methods, as the latter were presented in *La Géométrie*, may therefore be received without any sort of surprise. Whereas a complete demonstration of Newton's aversion against the method of analysis falls outside the scope of our investigation[8], his predilection for 'synthetic geometry' can be sufficiently stressed by the following:

> '[F]or anyone who examines the constructions of problems by the straight line and circle devised by the first geometers will readily perceive that geometry was contrived as a means of escaping the tediousness of calculation by the ready drawing of lines. The Ancients so assiduously distinguished them one from the other that they never introduced arithmetical terms into geometry' (Newton, 1972, V, p. 429).
>
> 'Present-day geometers indulge too much in speculation from equations. The simplicity of these is a consideration belonging to analysis: we are here occupied with composition, and laws are not to be laid down for composition from an analytical standpoint. Analysis guides us to the composition, but true composition is not achieved before it is freed from analysis. Let even the slightest trace of analysis be present in the composition and you will not yet have attained true composition. Composition is perfect in itself and shrinks from an admixture with analytical speculations' (Newton, 1971, IV, p. 477).

Newton's critical appraisal concerning the new mathematical method can be found permeated throughout his mathematical works. Yet, the cited passages are indicative of his reluctance towards the use of analytical method as a primary guide of reasoning and as a tool of scientific discovery. After all, whereas analysis deprives our senses from aesthetic elegance, one of the main reasons for its subordinating role consists in that it cannot sufficiently

[8] Guicciardini (2009) constitutes one of the most lucid and comprehensive inquiries on the subject of Newton's mathematical thought. Many of the views expressed here, own a lot to his inspiring work.

establish a certain ground for true laws. Newton rejected the use of Cartesian symbols for solving geometrical problems after having them reduced into equations and without providing any plausible geometrical demonstration in the end. Knowledge of a solution and its truth shouldn't be dependent on equations.

> 'To denote a root of a proposed equation geometrically, or insofar as a geometer should gather from the construction of an equation a solution of a kind propoundable and demonstrable without knowledge of the equation' (Newton, 1976, VII, p. 251).

In that way, old geometry was absorbed by analysis and demonstration (composition) would no longer be necessary. That approach actually paved the way for solutions in higher dimensional spaces, of which we have no perception.

Under this light, the phrase that Alfred Marshall has placed in a reply to Arthur Bowley, immediately after his famous six-step rule on how to use mathematics in economics, saying 'I believe to Newton's Principia methods because they carry so much of the ordinary mind with them' (Whittaker, 1996, p. 130) and which the editor of his correspondence has found to be 'obscure'[9], may no longer appear as enigmatic. In the Continent, even before Newton's death, the Newtonian approach was already becoming outdated. British scholars, on the other hand, bypassed new developments in mathematical thought with flamboyant indifference. Newton's influential legacy, though in retreat, was still present in mid-19th century Britain and this can be easily ascertained by the fierce criticism on the educational value of mathematics, unleashed by Scot philosopher William Hamilton.

Hamilton had foreseen the arrival of Continental or new method of mathematics to Britain since, after traveling to Germany, he had the opportunity to get in touch with the contemporary trends in philosophy and thought (Laita, 1979). His main arguments are based on the notion that mathematical reasoning is concerned with 'relations of quantities' and '[taking] no account of things but is conversant solely with certain images' instead of 'realities'. In addition, he turns next to the idea that mathematics is merely an 'explicative' art since it departs from a hypothetical definition instead of driving us towards it, and since its universality is 'not established on any a posteriori process'. Hence, it seems like our reasoning proceeds in mere forms devoid of particular meaning 'contemplating the general in the individual' (Hamilton, 1836, pp. 273-274).

> 'Mathematics, departing from certain hypotheses, and these hypotheses exclusively determining every movement of their procedure, and the images or the vicarious symbols about which they are conversant being clear and simple, the deductions of the sciences are apodictic or demonstrative; that is, the possibility of the contrary is, at every step, seen to be excluded in the very comprehension of the terms […]. The symbolic notation of arithmetic and algebra, is, that it has relieved the mind of all intellectual effort, by substituting a sign for a notion, and a mechanical for a mental process.' (Hamilton, 1836, p. 275).

The belief that algebraic symbols were capable of acquiring any potential meaning and imitating the essential features of anything our mind was able to reflect as quantity or degree of magnitude (with less and more), was gradually gaining recognition. For a long time,

[9] 'The aspect of Newton's methods that Marshall had in mind remains obscure' (Whittaker, 1990, p. 130).

mathematicians and philosophers were quite reluctant to go on naming symbols after moral sciences, but extending the ideas of analytical thought in the field of Political Economy was a matter of time. Not to mention that the need of mathematical analysis itself to extend onto other fields of knowledge seems to have troubled the mind of Descartes himself. As we learn from his biographer, Adrien Baillet, Descartes' 'own experience had convinced him a long time ago of the small benefit of mathematics, especially when men study it only for themselves, without applying it to other things' (Baillet, 1691, p. 111). A science of social affairs would certainly have sufficed for this purpose.

Jevons constitutes an exceptional example of those who rushed into embracing the idea of a world that is different from our own, in which other potential geometries would have been applicable. As he asserts, 'in other imaginary worlds the geometrical conditions of existence might be still more strange' (Jevons, 1871, p. 481). The main argument pertaining to the solid grounds of mathematical knowledge was, thus, reversed so that, even if it cannot be founded in the experience of our existent world then, an alternative can possibly be imagined. Contemplating upon several other different worlds, in which the analytical outcomes may be applied, would gradually lead us to a complete divorce from the traditional approaches of Whewell and Herschel. Truth was no longer to be founded within the objects of inquiry nor in experience, but in mental experimentation. 'Whatever other doctrines may be true, this doctrine of the purely empirical origin of geometrical truth is certainly false' (Jevons, 1871, p. 482). The terrain was open for analytic exploration and application and the new method constituted the ideological basis of neoclassical thought, meaning that it assisted the changing of perspective to accelerate.

5. Mathematical analysis, neoclassicism and Marshall

It was the wide acceptance and warm reception of the abstract visions of mathematical analysis and analytical geometry that helped neoclassical ideas to spread and dominate. In the Continent, Cournot (1838) and Gossen (1854) constitute two noticeable examples among the growing number of thinkers who took the first strikingly impressive steps towards the construction of a new theory of social conduct, following the application of those techniques to political economic issues. Both pioneering treatises were extraordinarily technical for their era whereas in both, the elimination of the adjective 'political' from the title of the old science was proposed as an almost technical prerequisite for complete abstraction from conflicting interests to be attained. Cournot focuses on the issue of bargaining under different market structures and is far more elegant in treating mathematical tools. Gossen's work, on the other hand, comprises a more ambitious attempt to incorporate the totality of human action, but his presentation is quite tedious and, in times, hard to follow. Nevertheless, the ardent admirers of mathematical analysis were primarily concerned with offering a satisfactory interpretation of their resulted outcomes, so as the various non-quantified concepts of moral status to be covered. Human will and self-interest were reduced into symbolic language.

Cournot claimed that the application of differential and integral calculus would purify political economy from its several incompatible theoretical standpoints and would transform it into a 'science of wealth'. The polemic against the old-fashioned theorists such as Adam Smith, was justified on the basis of misjudgment and prejudice, as a battle between the old tradition which refuses to withdraw and the new ways of thinking.

> 'I intend to apply to them [theories] the forms and symbols of mathematical analysis. This is a plan likely, I confess, to draw on me at the outset the condemnation of theorists of repute. With one accord they have set

> themselves against the use of mathematical forms, and it will doubtless be difficult to overcome to-day a prejudice which thinker, like Smith and other more modern writers, have contributed to strengthen' (Cournot, 1838, p. 2).

Any philosophical argument, or even Newton's critical stance, was seen as a parochial obstacle to the ongoing advancements of scientific progress. As far as Political Economy is concerned, Cournot continues, the prejudice lies on most peoples' false belief, that analysis inevitably leads to elementary algebra and numerical calculations. The object of this new method, he asserts, is intended towards the derivation of 'relations between magnitudes' that would be expressed as 'functions'. Henceforth, the answers to questions posed by political economy will be provided by 'that branch of analysis which comprises arbitrary functions' (Cournot, 1838, pp. 3-4). In other words, according to this new interpretation, any idea concerning quantity or measurement might as well be left aside, in order for the quantification problem of the moral aspects of economic theory to be overcome. In this extreme level of abstraction nothing would have been specific, nor number – designating even the slightest sense of quantity – neither function.

Hermann H. Gossen has also been occupied with the relation between mathematics and quantification as an impediment to considering issues of political economy. The fact that the idiosyncratic thinker lacked the capacity of Cournot in mastering the new techniques did not prevent him from dedicating considerable space in his *Entwickelung*, applying analytical techniques to moral issues. In this context, the laws governing individual action were examined in close relation to the aim of maximum lifetime pleasure, through the optimal allocation of several time-consuming enjoyments. To succeed in this, Gossen departs from the old geometrical rules and embraces the new method without questioning the validity of its knowledge. Accordingly, individual action was demonstrated in the new-fashioned way of diagrammatic analysis. It was believed that the use of symbols allowed him to reflect upon the unquantifiable nature of pleasure, by forming general expressions of the investigated laws that ultimately determine human behavior at all times and places.

> '[I]n geometry it is necessary to measure the actual dimensions of a given space in the real world in order to present its picture accurately. But such measuring is not required in order to elicit the laws of geometry since for this purpose it suffices to find, from the specific characteristics of space, means by which its parts can be mutually related. Similarly, in order to develop the laws governing enjoyment, it is not necessary to measure actually magnitudes of pleasure' (Gossen, 1854, p. 10).

Obviously, Gossen here refers to the new type of analytical geometry. In addition, he seems to suggest that analysis has assisted him in obtaining a complete solution of the long-lasting problem of value, the most vital issue in the field of political economy. By extending these rules to material as well as immaterial goods, he suggests, this science should no longer be called that anymore.

> 'The limitation [insofar] was imposed solely by the circumstance that it seemed impossible to formulate rules applicable above and beyond the material goods. The present conventional name of this science in no longer appropriate if we set aside this limitation and extend the purpose of this science to its real dimension – to help man obtain the greatest sum of pleasure during his life' (Gossen, 1854, pp. 38-39).

Hence, by generalising the features of analysis to immaterial and, consequently, non-quantified goods, and by abstracting from all other features that could determine human behavior, Gossen intended to transform political economy into a "science of pleasures" that would be deprived of issues of ideological dispute. Analysis was enthroned as the uncontested sovereign of human behavior, no matter whether this belief was founded on the quicksand of the unresolved issues of the nature of mathematical knowledge.

Walras was also in search for a new name of political economy. He, too, like Gossen, endorses the issue of scientific truth as the ultimate determinant of the field while, like Cournot, he goes against Smith's old-fashioned interpretation of the subject.

> 'The primary concern of the economist is not to provide a plentiful revenue for the people or to supply the State with an adequate income,[10] but to pursue and master purely scientific truths' (Walras, 1874, p. 52).

Apparently, the priority that was given to scientific techniques was total. Political economy ought, according to Walras, to no longer preserve the interests of the people but, on the contrary, it should pledge its belief to the scientific truths of analysis, no matter of its social outcomes. The new method substituted the view of empirical description and historical examination, but also changed the manner in which economic problems were posed. Indeed, the idea that mathematical analysis could have discovered problems that common sense cannot, has also been asserted by Edgeworth (1889, p. 547). He too upheld that the new method would help economists to 'discover loose quantitative relations of the form: x is greater or less than y' and not a particular numerical result or even a specific order of magnitude. The new theory 'assists us in conjecturing the direction and general character of the effect which changes […] The truth in its generality is more clearly contemplated by the aid of diagrams' (Edgeworth, 1889, pp. 539-540).

To our knowledge, Dardi (2006; 2016) gathers the only points of reference in which the method of analysis has been approached in sharp contrast to Alfred Marshall's tendency towards intuition and imagination. Within this revolutionary era of rapidly changing methods of economic reasoning, the late Victorian figure of Marshall stands alone by his peculiar approach towards the role of mathematics in economic theory. It should, therefore, come as no surprise that Marshall is conspicuous by his absence from the otherwise exhaustive treatise of Ingrao and Israel (1990) on equilibrium. Marshall's theory is solely mentioned in chapter 8 where Hick's contribution on dynamic equilibrium is presented. The authors admit that they 'shall take this opportunity to fill the gap in [their] treatment' (Ingrao and Israel, 1990, p. 217) and maintain that Marshall saw the 'impossibility of representing […] social systems through a broad and sophisticated mathematical apparatus' (Ingrao and Israel, 1990, p. 218). This is only partly true, however, since his hesitant stance is also attributed to the nature of mathematical analysis in itself and not just to its product, i.e. to the unrealistic occasion of static equilibrium and stationary state. This is also evident from the fact that in his investigation of the effects of possible changes in supply and demand, Marshall stresses that 'the unsatisfactory character of these results is partly due to the imperfections of our analytical methods' (Marshall, 1961, p. 809).

What is more, in Weintraub (2002, p. 24) we read that Marshall's own growing skepticism against the application of mathematical analysis to economics, constitutes a minor *Das Alfred Marshall Problem* for historians. However, to those aware of the disputes regarding the validity of mathematical truth, an invaluable comment made by Groenewegen

[10] Walras refers to Smith (1776, IV. Introduction).

may be of help in clearing out his seemingly misconceived or even 'schizoid', as Blaug has suggested (1962, pp. 398-9), attitude.

> 'Alfred Marshall was to apply his mathematics to economics with care, with caution and with a considerable degree of skill […] he preferred geometry more for this role than the terse language of algebra and the calculus' (Groenewegen, 1995, p. 94).

It should have been quite clear by now that the aforesaid difference between geometry and analysis is not just of appearance, but mainly of perspective. Marshall's *Principles* were written so that mathematical analysis would not interfere with the theoretical approach presented in the text, for he did not trust its long trains of reasoning imposed to economic theory (Marshall, 1961, p. 781). For example, in a letter to Flux, he added, 'my confidence in Cournot as an economist was shaken when I found that his mathematics [regarding Increasing Returns] led inevitably to things that do not exist and have no near relation to reality.' (Whittaker, 1996, letter n. 564) Additional evidence from his correspondence may also show that his conscious urge to favor the old techniques was based on the fact that that he had disavowed the emerging trend of analytical treatment of economic questions.

In replying to Foxwell, he writes that Mathematical Psychics (1881) 'on the whole disappointed' him (Whittaker, 1996, letter n. 98). whereas in another letter, in which Edgeworth himself pressures Marshall to offer him a straight answer as far as his opinion regarding geometry and analysis is concerned, the latter replies:

> ''[A]s regards the applications of geometrical rather than analytical reasoning I have not such decided views as you suppose.
>
> When tackling a new problem I generally use analysis, because it is handier. And in the book which I am just going to begin to write, I shall retain (in footnotes) a little mathematical analysis, for questions which I can't reduce under the grasp of curves. But – partly because, curves requires no special training, partly because they bear more obviously on the science of Statistics, I intent never to use analysis when I can use geometry.'' (Whittaker, 1996, letter n. 81)

The advantages of common-sense reasoning are here underlined while Marshall resists against the temptation offered by the extreme level of analytical abstraction to drive through answers posed by economic questions. He is clear about his preference, and the reasons that have enabled him to do so deviate substantially from the cruel explanation of Marshall's 'harboured ambition to be read by businessmen', given by Schumpeter (1954, p.722). Besides, from his reply to Walras, it is easily understood that his purpose in placing mathematics at the back was methodological. '[T]he right place for Mathematics in a treatise on Economics is the background. But I think it is most desirable that different seekers after truth should find different routes' (Whittaker, 1996, letter n. 273). Marshall's route towards truth, was certainly not passing through mathematical analysis.

6. Epilogue

Irving Fisher (1892) has carefully distinguished what he designates as the mathematical method of mainstream economic theory, which 'has reference to the use of symbols and their operations'. From his point of view, 'the introduction of mathematical method marks […] the entrance of political economy on a scientific era' (Fisher, 1892, pp. 105-109). Building up an economic theory can either be a process of the mind, of history, or of both. But the construction of neoclassical theory, as I have attempted to demonstrate, was particularly tailored under the ideological light of analytical mathematics.

Due to this ambition to understand the ideological aspects of mathematics in neoclassical economics, the distinction between synthetic and analytical reasoning was put forward. Mathematics, more than any other discipline, has influenced the modern state of economic thought. For instance, classical theories have, to a great extent been restated and interpreted in the new mode of analytical reasoning. But as has insofar been suggested, there is something essentially more than that. Mainstream economic theory does not constitute a mere restatement of the old and classical doctrines, but it is also the product of this same toolbox with which it is expressed. In that brief, mathematical analysis has paved a significant part of the long way towards the ideal formation of neoclassical economic thought, which in return, has been consequently adapted main aspects of the analytical perspective. To see that, the investigation of both histories of thought, economics and mathematics, must necessarily be studied synthetically and in collaboration.

My research agenda has drawn heavily on the idea that the foundations upon which human knowledge is grounded are highly contested. This fundamental acceptance left considerable room for fresh considerations and procures a great opportunity to investigate and revalue the history of mainstream economic theory as the ideological consequence of analytical mathematics. The history of analysis can bring to the fore several introspective aspects of interpreting society, which are found to be compatible with the main principles of neoclassical thought. In the narrow context of this paper the main source of ideology corresponds to the attempt of analytically interpreting the moral status of mathematical results. Be that as it may, this is just a step towards this more general purpose.

Acknowledgements

I am grateful to Sheila Dow, Brian O'Boyle and Rafael Galvão de Almeida for their helpful comments on the *Economic Thought* Open Peer Discussion forum

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SUGGESTED CITATION:

Missos, Vlassis (2020) 'Mathematical Analysis as a Source of Mainstream Economic Ideology' *Economic Thought*, 9.1, pp. 72-95. http://www.worldeconomicsassociation.org/files/journals/economicthought/WEA-ET-9-1-Missos.pdf