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Agents, Equations, and Economics

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Abstract

Critiques of Neoclassical Economics extend, unsurprisingly, to its mathematical structure. The discussion has largely focused on General Equilibrium Theory (GET), a formalism developed by Léon Walras over a century ago. Internally consistent, but highly unrealistic, GET lacks predictive power, and has been a historical failure. As an alternative, this article proposes a methodology largely developed by Gräbner et al. (2019), in which Agent-Based Models (ABMs) are linked with existing Equation-Based Models (EBMs) as a means of developing a more powerful formalism. The approach is illustrated by application to the Arrow-Debreu (AD) model of Neoclassical theory, and the Kuznets Curve of Developmental Economics. Broader implications for the social and natural sciences are briefly considered.

Key Words: economic methodology, Agent-based modeling, Equation-based modeling

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Introduction

Criticism of neoclassical economic theory has frequently extended to its mathematical structure. The focus of these critiques, now and for over a century, has been equilibrium theory: a formalism which specifies (among other properties) rational actors, full employment, and perfect competition (Turk, 2012). These idealized assumptions were defended by Milton Friedman (1966, orig. 1953) who argued that lack of realism in economic models was acceptable if the models had high predictive power. Apart from the question of realism in scientific models, to be evaluated briefly below, it was precisely in the realm of prediction that neoclassical economics was a historical failure. As Alberto Ruiz-Villaverde et al. (2019) have shown, the elegant mathematical edifice did not anticipate economic crises from its inception to the present day nor generate policies to effectively counteract them. In a somewhat earlier historical study, Nobel laureate Paul Krugman reached essentially the same verdict, diagnosing a long tradition of "mistaking beauty for truth" (Krugman, 2009). But while it is all well and good – indeed essential – to critique the formalism, it is of course a more difficult prospect to develop an alternative.

One possible solution is to use computational models – in particular, Agent-Based Models (ABMs) – to simulate and refine economic equations (Gräbner et al., 2019). (For a valuable discussion of a similar methodological situation in ecology, see DeAngelis and Yurek, 2015). The rationale is twofold: Agent-Based Models (ABMs), the focus of the present essay, are more methodologically compatible with Equation-Based Models (EBMs) than they may first appear. The Church-Turing Thesis (CTT) stipulates (essentially) that any real-world computation can be converted into a Turing machine computation (orig. Church, 1935). In addition, innovations in computational power – including, in particular, the merging of logic and memory functions, thereby largely surmounting the von Neumann "bottleneck" problem (Peper, 2017) – should facilitate the modelling of complex, multi-scale economic systems. David Colander speculates:

> maybe this future historian [of economics] will also point out that eventually, economics returned to its classical roots, but modernized them to take into account enormous advances in analytic and computational power that changed the way empirical data could be integrated with the mathematics of complex systems involving interacting strategic agents. (2011, p.20)

The essay begins with a defense of realism in economic model-building. We briefly contrast realist and non-realist philosophies of science, and suggest that in analyses of complex systems (e.g., economies, ecosystems), realist approaches provide greater information regarding the systems under study, and thus yield higher predictive power (Gräber et al., 2019; Maki, 2009). We then examine the properties of Agent-Based Models (ABMs) and Equation-Based Models (EBMs). We emphasize the empiricism of ABMs, which is consistent with a realist epistemology. Also, in accord with CTT, we underscore the mathematical compatibility of ABMs with EBMs. We then turn to applications in economics. Two pioneering studies are addressed in some detail: Albin and Foley (1992) utilized the complementarity of ABM and EBM modeling to elicit the properties of a decentralized Walrasian auction, and thereby refine a key component of Neoclassical Formalism (Arrow and Debreu, 1954); Gräbner et al. (2019) apply the dialog of models to an evaluation of the Kuznets Inverted U-curve Hypothesis that stipulates the relationship between economic growth and income inequality (Kuznets, 1955). Finally, we examine the broader implications for scientific investigation. Risking exaggeration, we suggest that the structured interplay of equation and simulation will have wide applicability, and may prove paradigmatically significant in the social and natural sciences.

Model-building: Realism, agents, equations

Nearly 70 years ago, Milton Friedman defended a non-realist philosophy of economic interpretation (Friedman, 1953). His *Essays in Positive Economics* and, in particular, the essay on "Positive Methodology" asserted an epistemology that was, and remains, quintessentially neoclassical. In a widely quoted passage, Friedman stated: "Truly important and significant hypotheses will be found to have 'assumptions' that are wildly inaccurate descriptive representations of reality, and, in general, the more significant the theory the more unrealistic the assumptions" (1966, orig. 1953, p.14). History has not been kind to this viewpoint, or to its larger context of neoclassical theory. However, the predictive failure of the framework did not lead, contra Friedman, to a questioning of its assumptions. Rather, the view was defended by an appeal to internal consistency: logico-mathematical agreement within its axiomatized structure. Gérard Debreu (1986, p.1265) was unequivocal: "According to the schema, an axiomatized theory has a mathematical form that is completely separated from its economic content. If one removes the economic interpretation of the primitive concepts, of the assumptions. . . its bare mathematical structure must still stand."

In marked contrast to this perspective, Leonardo Ivarola (2018) seeks to restore realism to economic theory. Ivarola critiques Friedman (1953) – and, implicitly, the "general impossibility of neoclassical economics" (Fine, 2011) – on both ontological and epistemological grounds. In the first of these, Friedman's notion of invariant relations among economic phenomena is replaced with an open-ended, decision-tree approach. Economic actors are, after all, humans; their economic decisions are shaped by their histories, their (emotionalized) personalities, and their sociocultural surroundings. Second, the flawed ontological assumption of invariance yielded a flawed epistemology that did not admit anomalies; as a result, and for nearly a century, the neoclassical framework has not predicted bubbles and crashes (Ruiz-Villaverde at al., 2019). Apart from theoretical critique, this history alone would suggest the need for a realistic, empirical economics.

Agent-based modeling (ABM) and equation-based modeling (EBM) can be creatively linked to address the conceptual flaws in neoclassical theory and, importantly, to develop a more realistic formalism. ABMs are simulations in which abstract entities (agents), governed by programmed rules, interact with one another in an artificial micro-world (Bruch and Atwell, 2015). Not infrequently, this process generates aggregate – emergent – properties unforeseen by the investigator. These, in turn, may contribute to the development of novel hypotheses. In contrast, and complementarily, EBMs typically model general systemic properties, and not individual features; EBMs are much less granular (Gräbner et al., 2019). More exactly, EBMs utilize ordinary differential equations (ODEs) to express the change in state of a many-component system over some specified period of time (Daun et al., 2008). That these highly contrastive approaches could be productively deployed together was recognized nearly 25 years ago. In population ecology, William Wilson (1998) demonstrated the complementarity of an ABM with an EBM in modeling the dynamics of predator-prey relations. The ABM simulated the aggregate effects of individual animal decisions, while the EBM expressed predator-prey population dynamics based on a traditional reaction-diffusion (RD) model of interacting chemical species (Kordo and Miura, 2010). Use of both approaches led to the mathematical refinement, and increased ecological realism, of small, dispersed populations, stochastic contact, and occasional extinctions.

Economic Applications: Addressing the Arrow-Debreu and Kuznets Models

In economics, as in population ecology, the interaction of an ABM with an EBM typically begins with the latter, which may be highly stylized (Gräbner et al., 2019). The equation is then converted into a preliminary ABM, which may be equally non-realistic. The prototype ABM is then tested to determine if it yields results equivalent to the EBM. What follows is a stepwise process – informed to no small extent by the creativity of the investigator – in which the ABM is enriched by empirical data, and the more "transparent" model may be expressed as a refined equation. The procedure is not without risk. As several advocates of the method have noted (e.g., Gräbner et al., 2019; Marilleau et al., 2018; Leombruni and Richardi, 2005), increasing an ABM's empirical richness through, for example, increased agent heterogeneity or greater number of mechanisms can cause the model to be unwieldy. However, as Gräbner et al. note, "starting with a simple, equation-based version and increasing the model's complexity stepwise helps to preserve its clarity. Also, the fundamental mechanisms of the model can usually still be communicated easily via precise equations" (Gräbner et al., 2019, p. 765).

An early application to economics addressed the Arrow-Debreu (AD) model, a core concept of neoclassical theory. The AD model, a formal proof of General Equilibrium Theory (GET), had posited an auction-like setting in which there exists some set of prices that would generate a balance of aggregate supply and demand (Arrow and Debreu, 1954). The proof has been controversial since its inception. Most notably, János Kornai, in an expansive critique, questioned the realism and scientific value of an "invisible hand" (Smith, 1776) – as in the AD model – that guided a capitalist economy toward equilibrium (Kornai, 1971; Schlefer, 2012). Contrasting with these critiques is the approach of Peter Albin and Duncan Foley, who view the AD proof as a starting point for devising a more powerful formalism (Albin and Foley, 1992). What would AD be if refined by realistic assumptions? Peter Albin and Duncan Foley (1992) developed an ABM in which the "auctioneer" was replaced by geographically dispersed agents in a system of decentralized exchange. Other key assumptions included costly advertising and bounded rationality. The changed assumptions resulted in unequal wealth endowments, with possible implications for mathematical modeling and economic policymaking.

In a more recent application, Gräbner et al. (2019) designed an ABM which simulated an enriched variation of the Kuznets Curve (Kuznets, 1955) and its underlying EBM, a formalism widely used in Development Economics. Simon Kuznets proposed that a rise in per capita income in a developing country was correlated with a sharp initial rise in economic inequality, which then plateaued, and ultimately declined, thus describing an inverted u-curve. (In an important variation – the Environmental Kuznets Curve (EKC) – directly relevant to sustainability, environmental deterioration is substituted for inequality. See Carson, 2010). The pattern was largely driven by rural-to-urban migration in an industrializing society. At the outset, financial opportunities were exploited only by a wealthy elite, but subsequently expanded to the larger society. Like its contemporary AD, the Kuznets model gave rise to an extensive critical literature (Lyubimov, 2017). The hypothesis, it has been widely noted, considered the West but not "the rest": it analyzed historical accounts of industrialization in Germany, Great Britain, and America, and then extrapolated – in an admittedly speculative spirit – to underdeveloped countries. Kuznets, to his credit, acknowledges this limitation, observing that the agrarian-to-industrial transformation may be markedly different in many developing countries, especially with regard to capital formation (Kuznets 1955, p. 26).

Is the Kuznets model valuable despite its flawed inception? The question motivated a recent study by Gräbner et al. (2019) utilizing a coupled ABM-EBM approach. Because Kuznets' original model (1955) did not contain equations, (although it included tabular data), Gräbner et al. developed a hypothetical EBM. The proposed mechanism underlying the inverted U curve was an initially wide wealth gap which impeded poor agents from transmitting resources (bequests) to their offspring. More exactly, Gräbner et al. specify an agent type $_{it}$ {p,r} where p designates poor and r designates rich, and each agent possesses an asset $h_{it}>1$which can produce a unique consumption good y. A fraction of the revenue deriving from the latter may be saved for bequests, designated by e. If $e<1$, the offspring will receive nothing, and the good y will be consumed. The persistence of the latter situation, as for example in an autocratic, exploitive economy, increases the threat of revolution, thereby generating reform, and a redistributive system. (For a broader discussion of the political economy of the Kuznets Curve, including – importantly – alternative trajectories (e.g. “autocratic disaster”) see Acemoglu and Robinson, 2002). The model was then incorporated into an ABM, which also included cultural regulations regarding marriage and inheritance, both of which are significant in the economics of developing societies. The resulting simulation found that “the time horizon of the Kuznets curve will vary with differences in initial distribution of wealth, differing degrees of social mobility, and alternative inheritance institutions” (Gräbner et al., 2019, p. 777).

Conclusion

Time has not been kind to Neoclassical Economics. From its inception to the present, the framework has failed to predict, or correct in a timely manner, economic dislocations in western capitalist societies. The consequent, and continuing, heterodox critique has frequently emphasized the formal axiomatic assumptions that lie at the heart of the paradigm. General Equilibrium Theory (GET) was, and has remained, a set of highly idealized constructs – e.g., rational agents, full employment, an optimized balance of supply and demand. This essay has suggested an alternative approach. We have proposed a computational strategy in which traditional economic equations can be coupled with agent-based models as a basis for developing a more scientifically powerful formalism. We then illustrated this viewpoint by sketching two pioneering studies. These applications showed how significant dynamic properties, which are concealed by an EBM, can be modelled by a realistic, empirically grounded ABM, and thereby furnish a basis for formulating a revised equation.

Importantly, the endeavor will involve significant challenges, some touching the basic relation of the scientist to reality. How, for example, does one choose the simulation approach to be coupled to a given equation? We have emphasized ABMs because of their descriptive richness, but other methods exist (e.g., cellular automata; mean-field game theory). Accordingly, a considerable effort is underway to devise a metalanguage which captures the optimally predictive relation between the scientist, the simulation, and the real world referent. A recent example along these lines is the framework developed by Gräbner (2018): from an infinite number of referent (target) properties, a selected set is instantiated by software agents, to which they are conceptually linked by a “key” (like the legend of a map). Such frameworks, as Vallverdu (2014) suggests, are simultaneously retrospective and anticipatory. Their notion of a cognitive and instrumental editing of reality has a distant antecedent in Kant, while the computational properties of interacting artificial agents are introducing a changed understanding of scientific experimentation. On the latter aspect, Schiaffonati (2016) has noted, a computational investigation will often generate unanticipated results. The epistemology is thereby shifting. It is a posteriori rather than a priori.

We conclude by asking if there are broader implications for the method outlined here. Does the "dialog of models" have significance not only for economic analyses, but for the social sciences generally, and indeed for any natural science that examines large, complex systems (e.g. population ecology and molecular cell biology)? Donald DeAngelis and Simeon Yurek (2015), influenced by the modeling of nonlinear dynamics in ecosystems, have recently considered this question. Defending the importance of computational models such as ABMs and cellular automata in the study of complexity, and presenting an argument largely consistent with the essay, they further note: "[S]cience may be moving into a period where equations do not play the central role in describing dynamic systems that they have played in the last 300 years" (p.3857). Perhaps they are right, but this may be a bridge too far. We would suggest that equations will probably never be less important – in economics, ecology, or any other science – than computational modeling. The approaches are interdependent, equally essential, and will share center stage. The dual strategy – a methodological pluralism – will hopefully promote a more realistic understanding of the dynamics of complex systems, as well as increased precision in prediction and application.

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