Mathematics, Science and the Cambridge Tradition

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Abstract

In this paper the use of mathematics in economics will be discussed, by comparing two approaches to mathematics, a Cartesian approach, and a Newtonian approach. I will argue that while mainstream economics is underpinned by a Cartesian approach which led to a divorce between mathematics and reality, the contributions of key authors of the Cambridge tradition, like Marshall, Keynes and Sraffa, are characterised by a Newtonian approach to mathematics, where mathematics is aimed at a study of reality. Marshall was influenced by the Newtonian approach that still characterised many aspects of the Cambridge Mathematical Tripos, where the emphasis was on geometrical and mechanical examples rather than on symbolic (Cartesian) mathematics. Keynes, who criticised (Cartesian) symbolic mathematics, was indeed an admirer of Newton and of his method. Sraffa's mathematical constructions are also in line with the Newtonian approach where arithmetic and geometry were strictly separated, since Sraffa's mathematical constructions typically use arithmetic without engaging in the mixture between geometry and arithmetic that occurs in the Cartesian approach.

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Introduction

This article aims to establish three theses. The first thesis is that since the modern age started there has been a tension between a Cartesian approach to mathematics, and a Newtonian approach to mathematics. The second thesis is that contemporary mainstream economics is characterised by a Cartesian approach to mathematics, which indeed became the dominant approach to mathematics, leaving the Newtonian approach in the shadow. The third thesis is that central authors of the Cambridge tradition in economics, like Alfred Marshall, John Maynard Keynes, and Piero Sraffa, are closer to the Newtonian conception of mathematics than to the Cartesian conception of mathematics.

I shall start by explaining the first thesis, concerning the distinction between the Cartesian approach to mathematics, where geometry and arithmetic (and thus algebra too) are not carefully separated, and the Newtonian approach to mathematics, where geometry and arithmetic (and thus algebra too) are kept carefully separated. I will then move to the second thesis showing that mainstream economics is characterised by a Cartesian algebraic approach, and then defend the third thesis, showing how a Newtonian approach, in which there is a strong interplay between mathematics and reality, was once dominant within the Cambridge tradition.

This interplay, and the search for mathematical truths that can be known with certainty, led to a tension between a given philosophical conception of reality, and the search for certainty through mathematical methods, which characterises the work of Marshall (and exists also in Newton's work). This tension springs from the incompatibility between an ontological conception of reality where the latter constitutes an open system, and the limitations that mathematical methods which aim at certainty face in such situations.

Mainstream economists eliminated this tension by embracing a conception of mathematics where the latter is divorced from reality (following a Cartesian conception, which in mathematics culminated in Hilbert's program and the Bourbaki school, and in economics led to the general equilibrium analysis of Kenneth Arrow, Gérard Debreu and Lionel McKenzie). Keynes resolved this tension by focusing on the open nature of social and economic reality. This led Keynes to replace the search for certainty in logical relations for a conception where statements can be known with a given degree of probability (given the available evidence), and where the probability distribution may not even be represented numerically (unless in case of exhaustive, exclusive and equal-probable alternatives). In Keynes' conception, the open nature of social and economic reality means that we are typically in a situation of uncertainty, and not of certainty, as will be argued here.

Sraffa also provides an example of the Newtonian approach, since one of the characteristics of the latter is a strict separation between geometry on the one hand, and arithmetic and algebra on the other hand. This separation does not exist in the Cartesian approach. Sraffa's mathematical constructions, by drawing upon arithmetic without simultaneously drawing upon geometrical notions like a continuous line full of real numbers when showing the existence of (or indeed constructing) the main notions used in his work, maintains the Newtonian emphasis on the separation between geometry and arithmetic.

Cartesian and Newtonian Approaches to Mathematics

As Immanuel Kant argued, our pure intuition of space provides us the subjective conditions of possibility for our representation of geometrical figures, while our intuition of time, and of the process of counting through time, brings us the series of natural numbers. Hence, spatial figures are our primary elements in geometry, while natural numbers are our primary elements in arithmetic, and also in algebra. Indeed, algebra can be seen as a development of arithmetic that emerges when arithmetic operations are applied to both sides of an equation, and thus an arithmetical operation performed on one side of the equation has also to be applied to the other side of the equation, balancing the equation by restoring the equality between the two sides.

The separation between geometry and arithmetic (or algebra), that existed until Descartes, can then be seen as a natural consequence of the fact that these two fields are built upon these very different notions, geometrical figures on the one hand, and natural numbers on the other hand, which according to Kant come from different sources of our intuition (namely, space and time, respectively).

But when Descartes introduces numbered axes into geometry (the famous Cartesian coordinates) we have a mixture of these two fields, which emerges in the Cartesian approach, and was criticised by Newton. Michael Atiyah argues that Newton and Descartes follow two different traditions within mathematics where the relative weight of each of these two fields, geometry and algebra, is very different:

> "Geometry and algebra are the two formal pillars of mathematics, they are both very ancient. Geometry goes back to the Greeks, and before; algebra goes back to the Arabs and Indians, so they have both been fundamental to mathematics, but they have

had an uneasy relationship. Let me start with the history of the subject. Euclidean geometry is the prime example of a mathematical theory and it was firmly geometrical, until the introduction by Descartes of algebraic co-ordinates, in what we now call the Cartesian plane. That was an attempt to reduce geometrical thinking to algebraic manipulation. This was of course a big breakthrough or a big attack on geometry from the side of the algebraists. If you compare in analysis the work of Newton and Leibniz, they belong to different traditions, Newton was fundamentally a geometer, Leibniz was fundamentally an algebraist." (Atiyah 2005: 655)

Newton thought that geometry, the study of figures produced by mechanical constructions, should be kept separate from arithmetic, which is the study of numbers and the operations associated with it. Thus, when Descartes introduces algebraic coordinates into geometry, he is mixing the two fields that Newton thought should be kept separate.

Indeed, the implications of the Cartesian method are very deep, and raise enormous issues, which are still being discussed today. The Cartesian axes presuppose the idea of a line where a real number can be attributed to every point of the line. We are in the habit of calling this line the "real line", which we take to be a continuous line, since no space is left between the points that constitute it, and thus those infinite points of the real line are "already there", rather than being constructed through arithmetical approximation.

However, it was not through this notion of a "real line" that we first reached the set of real numbers. Rather, it was from arithmetic that we got there. For example, if we start from a purely arithmetical procedure, performing various operations such as addition, subtraction, multiplication and division of natural numbers, we reach the set of rational numbers, which includes not only integers, but also the fractions of one integer by another. By engaging in more operations we may also ask ourselves what is the number that multiplied by itself will lead to a given number, for example to the number two. The number that satisfies such a condition is, of course, what we call the square root of two. But we will then find that this number, the square root of two, is not part of the set of rational numbers, since we cannot find any fraction of natural numbers that is exactly equal to it. Thus we have found an irrational number. It is through the performance of arithmetical or algebraic operations like this that we reach irrational numbers, and add them to rational numbers in order to reach the set of real numbers.

But from an arithmetical perspective, it would be misleading to take the set of real numbers as something that is "already there". Rather, those numbers are obtained through algorithmic procedures of approximation, through operations on natural numbers. We cannot know exactly the square root of two in any way other than computing further decimal cases through arithmetical operations of approximation. However, when we mix arithmetic with geometry as Descartes did, we are easily inclined to accept the idea that the irrational numbers are "already there" in the "real line", since we see the line as a continuous line, taking however the notion of continuity from the geometrical line, not from the arithmetic where the numbers (that we place in the geometrical line) came from.

In short, since the emergence of the Cartesian approach where geometry and arithmetic are not separated, we are in the habit of seeing the real line as being made up of real numbers, which include also irrational numbers like the square root of two. But how can we think of such numbers as being already there in the real line, if we cannot identify any of these numbers outside the context of an arithmetical procedure of approximation? A solution that emerged in the Cartesian approach is to think of points in the "real line" as possibilities, rather than an actual number. This leads to the notion of a variable, that is, a possibility of various numbers, rather than an actual number, which was developed by

Descartes. A variable starts indeed to be seen as an existing entity, while before Descartes only actual numbers (and not possible numbers such as a variable) would be seen as a mathematical object.

The differences between Newton and Leibniz regarding differential calculus are also related to the distinction between geometry and algebra. Leibniz, who was in the tradition of Descartes, had the aim of ‘formalising the whole of mathematics, turning it into a big algebraic machine’ (Atiyah 2005: 655). Newton saw essentially two problems with Leibniz’s approach. First, the use of infinitesimals in Leibniz’s differential calculus. An infinitely small quantity is not something that we can conceive in our mind as an existing entity. Rather, it is the result of an algorithmic procedure undertaken indefinitely (effectively, “infinite” means something that is not finite, that does not end, and is thus “indefinite”).

In fact, Leibniz himself emphasised how this algorithmic operation was the more important part of his calculus, since he wanted to keep mathematics free from philosophical problems. Infinitesimals would thus be not an entity, but rather the shorthand for an algorithmic operation. But because Leibniz and his followers were drawing upon the Cartesian tradition where geometry and algebra are not separated, it became easy to interpret the “infinitesimally small quantity” as a Cartesian variable in a “real line”, and thus as an entity, rather than as the name of an operation, as probably Leibniz would have wanted.

This leads to the second problem with Leibniz’s approach. This problem springs from the fact that even if Leibniz could avoid the mixture of geometry and algebra by taking his calculus to be an algorithmic procedure where infinitesimals are only a name for an operation rather than a variable, for Newton an algorithmic procedure is only a method to find truth, not a rigorous demonstration that provides a construction of the problem. That is, for Newton, only through geometry can we “show” truth, and algebra is only an instrument to get close to truth through a process of approximation. And Leibniz’s infinitesimals, either interpreted as an entity or as an expression for abbreviating reasonings, are never an exact finite quantity. And thus, even assuming that Leibniz could successfully separate arithmetic from geometry avoiding the problem of whether infinitesimals are a real entity in a line, he was still using the less appropriate method: arithmetic, rather than geometry. For arithmetic can lead to the solution only through successive approximation, while geometry would provide an exact demonstration that does not work through particular symbols, as Leibniz’s calculus, but through areas and lines without resorting to symbolic calculus that can only proceed through analytical approximation.

There are other reasons for Newton’s position. The fact that geometry provides a more natural method, that can be used more intuitively than symbolic calculus was also important for Newton, since for him mathematics should provide a description of Nature that could also be easily explained to the non-mathematician. Note that Newton valued not only the fact that a demonstration could be more accessible to the non-mathematician through geometry rather than through symbolic calculus, but also the fact that geometry can more easily describe Nature, where we find continuous motion as in the mechanical process of drawing geometrical figures.

Furthermore, it is certainly true that given Newton’s voluntarist philosophy, in which the universe is always open to the intervention of God, he would not like the idea of turning the world (or God) into an algorithmic machine. But this latter issue is a side issue, since Leibniz did not want to mix philosophy with mathematics, and Newton’s own criticism of Leibniz focuses on the mathematics too, rather than on the philosophical divergences.

From Algorithmic Arithmetic to Set Theory

While the Newtonian approach remained the dominant approach in Cambridge and England after Newton's death, the European continent was dominated by the Cartesian perspective. Within the European continent, the tension which resulted from the Cartesian mixture between geometry and algebra remained. Surely, many mathematicians, from Carl Friedrich Gauss (an admirer of Newton) to Leopold Kronecker, were able to keep a rigorous separation between geometry and arithmetic. But the use of Cartesian coordinates to represent geometrical figures that became widespread contributed to blur the distinction between the strictly arithmetic operation of approximation of any real number by a rational number, and the strictly geometrical representation of a continuous line.

But once we juxtapose the arithmetical operations over natural numbers with the geometrical notion of a line, as the Cartesian coordinate axes do, a problem emerges. Because strictly arithmetical operations over natural numbers can only lead to a rational number, and rational numbers are only a subset of the real numbers that constitute the continuous real line, when we juxtapose the smaller set of rational numbers over the larger set or real numbers that constitutes the "real line", there will be "holes" left unfilled in the real line. For although we can approximate a real number by a rational number as close as we want through an algorithmic procedure, we will never be able to reach the real number in a finite number of steps.

Richard Dedekind suggested that we fill those "holes" through a procedure known as "Dedekind cut". A Dedekind cut provides a partition of rational numbers into two non-empty subsets. The cut can be a rational number that belongs to one of the subsets, and sections the set into two subsets. But imagine that one of these two subsets is not complete, and has an open interval, such that it may have no rational number as its lowest element (if it is the subset of rational numbers higher than the cut) or no rational number as its highest element (if it is the subset of rational numbers lower than the cut). In any of those situations, the "cut" which divides the two non-empty subsets of rational numbers must be an irrational number. Thus, according to Dedekind, the cut creates a new irrational number, which is completely defined by the Dedekind cut. Since for every Dedekind cut we have either a rational number or an irrational number, we may fill the real line through these cuts with the rational numbers and irrational numbers that were missing, and achieve a continuous "real line" with no "holes".

This procedure is clearly grounded in the Cartesian mixture between geometry and arithmetic, since the idea of cutting a line appeals to geometrical intuition. The cuts which are being used to fill the real line with numbers are thus inspired in our geometrical intuition, but are employed in order to fill the gaps of the real line where a rational number could not be placed.

Dedekind's procedure allows one to simply posit the existence of any given real number, defined by the cut, rather than indefinitely approximating it through arithmetical operations on natural numbers. But once we are allowed to simply posit the existence of a mathematical entity, rather than obtaining it through indefinite arithmetical approximation, new perspectives emerge for mathematics.

In fact, if an indefinite algorithm can be substituted by an existing entity, it is not only the infinite decimal cases of a real number that can be taken as given (rather than approximated through arithmetical operations on natural numbers), but also the infinite itself that can be taken as given. That is, rather than focusing on the indefinite procedure of counting, which generates as many natural numbers as we want, we can simply take the infinite set of natural numbers as an existing entity. And the same can be said of the set of even numbers, or rational numbers, or real numbers. This is what Georg Cantor did. But Cantor saw that once we take those sets as given, all the infinite numbers they contain must be "already there". This led to an approach that treats infinite sequences as if they were finite.

Note that if we choose to count only a finite quantity, for example until ten, we can see that the set of natural numbers obtained is constituted by ten elements, while the set of even numbers obtained will be constituted by five elements. Thus, when dealing with these two finite sets, we can say that the latter set is smaller than the former set. But once we take the infinite number of members of those sets to be already given too, we will then see that the set of even numbers also contains holes that can be filled with other natural numbers (namely the odd numbers), that the set of natural numbers has holes that can be filled with other rational numbers (namely the fractional numbers), and that the set of rational numbers has holes that can be filled with other real numbers (namely the irrational numbers). In other words, we can establish a correspondence between elements of those sets and see that one set contains all elements of the other, but not vice-versa. Thus Cantor concludes that there are infinities of different cardinalities.

We would not be led to this conclusion if we had simply taken infinity to mean an indefinite algorithm of construction of numbers, in which case we would not compare algorithms which operate indefinitely on different sets of numbers. But once we take mathematical entities like numbers to be "already there", for example in a "real line" (a geometrical line populated by numbers), we will reach this type of conclusion.

In fact, the Cartesian method is at the origin of this type of conclusion, since it was the Cartesian method which led us to posit the existence of various points that can be numerically juxtaposed to a geometrical line, thus bringing in the notion of a variable, which leads us to posit the existence of an arbitrary number, rather than to construct an actual number through arithmetical approximation.

The assumption of the existence of arbitrary numbers started to play an important role in mathematical proofs, for example Bolzano's proof of the result that if a continuous function has a negative image for a given number, and a positive image for another number, and the function is continuous between those two numbers, then its image must have passed through zero at some point, or for some number (notice the mixture between arithmetic and geometry used in the proof through simultaneous references to numbers, and to geometrical points, or to the intuition of a geometrical line crossing the Cartesian axis of the abscissas). These types of existence proofs allow us to proceed by assuming arbitrary numbers or variables (and mixing geometry with arithmetic) without first constructing the actual numbers we would need for the proof.

Before the end of the nineteenth century these ideas were not fully clear. Remember that Leibniz seemed to see his symbols as operations too, rather than actual entities, while nevertheless calling them infinitesimals and treating them as variables. But after Cantor, symbols are already being explicitly used as actual entities, rather than as shorthand for a process of construction. Cantor's set theory is a development enabled by the Cartesian method, which would however be seen as seriously misguided if we adopt the Newtonian strict separation between geometry and arithmetic.

Of course, the mathematicians who followed the Cartesian approach tried to establish foundations for their conclusions which would go beyond the contradictory ideas that emerge when mixing geometry and arithmetic. In the initial approach to set theory adopted by Cantor, sets were treated intuitively as arbitrary collections of elements, which could be defined extensively (in terms of their elements) and according to a given abstract property. But this led to paradoxes, such as the one found by Bertrand Russell, when noting the consequences of defining the set that has the property of having as its elements all the sets that are not a member of themselves. When asking if this set is a member of itself, we reach the contradiction that if it is not a member of itself, then it must be a member of itself, and vice-versa. Russell's paradox led to an abandonment of a more intuitive approach to set theory, which also had implications for the way in which we approach the real line.

The Cartesian procedure presupposes that the Cartesian axis is ordered, and indeed there have been attempts to establish the continuity of the real numbers over the real line without resorting to intuitive geometrical analogies that are foreign to arithmetic and algebra, for example by assuming that the set of real numbers is well-ordered, and thus we can order it to obtain a continuum, hence leaving no "holes" in the middle. Ernst Zermelo, who had also found independently the paradox formulated by Russell, provided a theorem according to which the elements of a set can be well-ordered, and thus the real numbers could be well-ordered too, thus apparently solving the problem of the rigorous definition of a "real line" which arises from the Cartesian methodology, without however having to appeal to geometrical intuition.

Cantor found that the assumption that the elements of a set can be ordered is indeed essential. Julius König, however, argued that it is not possible to order the set of real numbers. König noted that the set of numbers which can be finitely constructed is a subset of the set of real numbers. But if we could order the set of real numbers, there would be a first number which is not finitely definable within this ordering. But the rule "find the first number which is not finitely definable" provides a way to define this very number in a finite way (in a finite number of steps), and thus the number was finitely definable after all, leading to a contradiction.

Besides König, other mathematicians who opposed Cantorian set theory were Henri Poincaré and Émile Borel, but the opposition had started earlier with Kronecker, who famously argued that God created only the natural numbers, and everything else is constructed by human beings. Kronecker's claim entails that real numbers, and indeed all numbers other than natural numbers, are not "already there", and must rather be constructed through algorithmic processes of approximation. L. E. J. Brouwer is usually seen as the founder of the constructivist approach which already underpinned Kronecker's contribution, However, with Brouwer the term "intuitionism" is also often used, due to the fact that the constructivist approach starts from entities which are claimed to exist in our pure intuition (following Kant). These include geometrical figures, (which come from our intuition of space) and natural numbers (which arise through our intuition of time).

Brouwer's approach to mathematics influenced the philosophy of mathematics developed by Ludwig Wittgenstein. Wittgenstein was highly critical of Cantor's set theory precisely because it posits as existing entities what Wittgenstein sees as a rule of indefinite approximation. Mathematics, for Wittgenstein, is characterised by a given rule that we apply. Thus, for Wittgenstein arithmetic is, just like geometry, a process of construction according to a rule. Arithmetic simply operates with whatever is posited at the beginning of the operation. Thus if we start our operations with natural numbers, we will not end up with real numbers, but only with rational approximations to it, because no rule can lead from rational numbers to real numbers.

Cantor's set theory revolutionised mathematics, and provided a development of the Cartesian mixture between geometry and algebra. One way out of the problems raised by the Cartesian methodology would be the one suggested by Newton, which consisted in returning to the ancient mathematical tradition where geometry and arithmetic were fully separated. But another way out, developed first by Dedekind and afterwards by Cantor and Zermelo, consisted in finishing Descartes numbering of the real line, filling in the places that could not be filled by operations on natural numbers. This is what Dedekind, Cantor and Zermelo did. And because their approach simply posited mathematical entities, mathematics was already complete with its own entities which only had to be found (rather than being a permanent process of construction), and had no need to be compared with an external reality.

The term "Platonism" is often used to denote an approach where mathematical entities like those are posited. However, the term is unfortunate, since unlike this contemporary mathematical "Platonism", Plato and the Platonists did not accept any mathematical entities into the realm of Platonic ideas other than geometrical figures and natural numbers which are, furthermore, kept separated, since Plato and the Platonists accepted a strict separation between geometry and arithmetic.

It is indeed this old Greek tradition that Newton is trying to recover. And in this ancient tradition that Newton recovers, geometry and arithmetic are kept separated, and are also separately applied to the study of Nature. In the Newtonian approach, geometry and arithmetic are two separate methods of construction according to a rule (to use Wittgenstein's term), which remain mere tautologies until they are applied to a given reality, such as the study of Nature.

In the Cartesian approach, in contrast, mathematics is not only a method of (geometrical or arithmetical) construction, as in the Newtonian approach, but it is a study of existing mathematical entities, which are generated by the mixture between geometry and algebra. But since mathematics already possesses its own entities in the Cartesian approach, there is a tendency to feel that there is no need of applying it to further external entities, such as an external reality. Just like Descartes argued that philosophy must proceed through pure reasoning which is not connected to empirical reality, so did Cartesian mathematics develop in a formalist fashion that is separated from empirical reality, in contrast with the Newtonian approach where mathematics was a study of Nature. Indeed, the term "formalism" is often employed to denote mathematical developments of the Cartesian approach.

The Cartesian and Newtonian Approaches within Economics

The Newtonian approach is often misunderstood, and taken to be formalistic too. But as Tony Lawson (2003: 263-264) notes, this happens because the successes of Newton's mathematics were interpreted in France in Cartesian terms, and not as evidence that mathematics can be most useful when it takes into account the nature of the reality under study. The differences between the Scottish Enlightenment, where Newtonianism was interpreted as a study of Nature which is concerned with empirical reality, and the French Enlightenment, where Newtonianism was interpreted as a reduction of Nature to Cartesian mathematics, are symptomatic of this difference. The Cartesian approach to mathematics that prevailed in France shifted the attention towards the mathematical formal structure of Newton's theory, and neglected the way in which the study of Nature, of an external reality, influenced the development of such a theory.

The French Bourbaki school, which Atiyah (2005: 655) identifies as the twentieth century heir of the Cartesian approach, is indeed an example of this type of approach, which inspired central contributions to mainstream economics like Gérard Debreu's (1959), in a context where economics (like Cartesian mathematics), became concerned with possible realities, and not with actual reality itself – see Dow (2003) or Lawson (2003: 271-273). In order to understand the use of mathematics in mainstream economics, it is essential to understand that it was the Cartesian approach to mathematics that became dominant within mainstream economics, leading to a conception where reality is neglected.

This was the Cartesian approach to mathematics towards which Jean-Baptiste Say was hostile. And it was the approach which ultimately accepted the work of Léon Walras and the marginalists. But within this approach, there was no attempt to combine marginalism with classical political economy as Alfred Marshall arguably attempted to do. In fact, classical political economy was rejected, leading to the uncritical use of mathematico-deductivist methods that took place within mainstream economics in the twentieth century, as we shall see.

It is thus no surprise that contributions characterised by a mathematico-deductivist approach, of authors like Debreu (1959), are considered as the more elaborate and admirable achievement of economic theory within mainstream economics. Within this approach, many theorems rely on the continuity of the "real line" established by Dedekind, Cantor and Zermelo, especially within fixed point theorems where the continuity of the real line is assumed, rather than (algorithmically and arithmetically) constructed (and we find a reliance on the existence of arbitrary numbers, i.e. variables, rather than constructing the actual numbers needed for the proof).

While mainstream economics is best characterised in terms of a mathematico-deductivist methodology (see Lawson, 2003), it is true that most mainstream economics subscribes also to the theoretical principles of marginalism, which pioneered the use of mathematical methods in economics. And marginalism is often identified with what is today called *neoclassical economics*. This identification, together with the fact that there is much overlap between mainstream and marginalism, leads many to refer to "neoclassical economics" as the dominant economic perspective.

However, this use of the term "neoclassical economics" is a misleading one. After the marginalist revolution, Marshall (1890) developed a conception which seemed to many to be a way to make marginalist theory compatible with classical political economy, leading to an approach which was thus termed by Thorstein Veblen (1900) as neo*classical*, in order to distinguish it from the approach of Carl Menger (which inspired Austrian economics). In fact, Keynes (1936) considered Marshall's "neoclassicism" to be a continuation of classical economics. In so doing, Marshall initiated the Cambridge economic tradition, shaped by his contribution (see Harcourt, 2003), while Marshall's (1890) *Principles of Economics* became the canonical economics textbook not only in Cambridge, but in many other economics departments.

The fact that Marshall attempted to establish continuity with Smith, Ricardo and John Stuart Mill does not mean that he succeeded. In fact, as Pierangelo Garegnani (1998, 2005) explains, Smith and Ricardo did not analyse value in terms of the interaction between supply and demand, as Mill and Marshall did. It was Piero Sraffa who, after having criticised Marshall's approach, which had been developed by Pigou – Sraffa (1925, 1926) – recovered the classical theory of value of Smith, Ricardo and Marx, beginning to develop it after the end of 1927, in a project which led to Sraffa's (1960) revival of classical political economy.

Most of Sraffa's work remains unpublished. In fact, as Annalisa Rosselli (2005: 405) notes, Sraffa writes, in a letter to Charles Parrish Blitch (dated October 6, 1975), that "in economic theory the conclusions are sometimes less interesting than the route by which they are reached", which signals how Sraffa was more concerned with the process of discovery rather than with the publication of results. But if we take Smith and Ricardo as the key authors of classical political economy, as Marx did (rather than Stuart Mill's version of it), then it is Sraffa (and not Marshall), who really engages in a continuation of classical political economy.

The Cambridge economic tradition is divided into various streams, including not only the Marshallian stream, but also the Keynesian and Sraffian streams, which emerged with the critique of Marshall undertaken by Keynes and Sraffa. But there is one aspect of Marshall's work which remained throughout the Cambridge economic tradition, namely Marshall's realist approach, which contrasts with Walras's use of mathematics. As Joseph Schumpeter argues:

> "Just as Walras, more than any other of the leaders, was bent on scraping off everything he did not consider essential to his theoretical schema, so Marshall, following the English tradition, was bent on salvaging every bit of real life he could possibly leave in." (Schumpeter 1994: 974)

This difference between Walras and Marshall shows itself in the fact that while Marshall thought that mathematical analysis should be left out of the main text of a book (as he did in his work), mainstream economics, following the Walrasian approach, evolved in a way where mathematical analysis became the central aspect of any mainstream economics textbook.

In fact, the use of mathematics in Marshall, and in the Cambridge tradition, is very different from the mainstream (Cartesian) use of mathematics. Marshall and the Cambridge tradition were much influenced by the Newtonian approach to mathematics. It is true that after 1815 at least, the Cartesian approach was gaining ground in Cambridge too, and Marshall himself uses an approach to differential calculus that is highly influenced by the Leibnizian (Cartesian) approach. But as Simon Cook (2009) explains, the Cambridge Mathematical Tripos that Marshall undertook still contained many elements which remained from the Newtonian approach, such as an emphasis on geometrical and mechanical problems, rather than on symbolic algebra. This is not unrelated to the fact that, for many years, Cambridge and England followed the Newtonian approach, while the Cartesian approach was already dominant in the European continent.

The Search for Certainty

As noted above, Newton was concerned with providing a foundation for science which takes into account the nature of reality, which was also a central concern for Marshall. However, Newton was also very concerned with achieving certainty. As Guicciardini (2006: 1736) explains, “from the early 1670s [Newton] expressed his distaste for the probabilism and hypotheticism that was characteristic of natural philosophy”, since mathematics should provide certainty to natural philosophy.

However, as Guicciardini (2006: 1736) also notes, there was a tension in Newton’s perspective, for while “Descartes was the champion of an impious mechanistic philosophy (...), Newton conceived himself as a restorer of an ancient, forgotten philosophy according to which nature is always open to the providential intervention of God”, and thus, this “led Newton into a condition of strain, since his philosophical values were at odds with his mathematical practice, which was innovative, symbolical, and – pace Newton – deeply Cartesian.”

Guicciardini (2006: 1736-1737) continues:

> “Several hitherto unexplained aspects of Newton’s mathematical work are related to this condition of stress and strain that characterizes his thoughts on mathematical method. Why did Newton fail to print his method of series and fluxions before the inception of the priority dispute with Leibniz? Why did he hide his competence in quadratures when writing the *Principia*, which are written mostly in geometrical style? Even though there is no single answer to these vexed questions, I believe that Newton’s conviction that the analytical symbolical method is only a heuristic tool, not ‘worthy of public utterance’, can in part explain a policy of publication which was to have momentous consequences in the polemic with Leibniz.”

There is an interesting similarity between the explanation that Guicciardini (2006, 2009) gives for Newton’s postponement of the publication of his work, and the explanation that Stephen Pratten (1998) provides for Marshall’s failure to finish the second volume of his *Principles*. Pratten (1998) argues that it was the tension between Marshall’s vision of reality, and his use of a mathematical method which was inappropriate for analysing such reality, that prevented Marshall from achieving a satisfactory second volume, that would reconcile the statical method used in the first volume with the dynamical approach that was to be developed in the second volume – see also Neil Hart (2004) for an explanation of this tension in terms of the relation between theory and history in Marshall.

The tension in Marshall's approach is often connected to the fact that while Newtonian physics inspired much of the first volume of the *Principles*, Marshall notes that the Mecca of every economist lies in biology. It may however also be the case that for Marshall mathematical physics and biology were not so far apart, for in the mathematical appendix (note XI) of the Principles Marshall writes:

> "There is more than a superficial connection between the advance made by the applications of the differential calculus to physics at the end of the eighteenth century and the beginning of the nineteenth, and the rise of the theory of evolution. In sociology as well as in biology we are learning to watch the accumulated effects of forces which, though weak at first, get greater strength from the growth of their own effects; and the universal form, of which every such fact is a special embodiment, is Taylor's Theorem" (Marshall 1920[1890]: 694)

Taylor's theorem is named after Brook Taylor, who was at Cambridge at the same time as Newton (and in fact was part of the committee that decided Newton's dispute with Leibniz). This cannot be seen however as further evidence of the influence of the Newtonian approach in Marshall, since this theorem is used by mathematicians of both (Newtonian and Cartesian) approaches, and indeed Marshall often uses Leibniz's notation. Marshall follows the broad realist approach that characterises the Newtonian approach, but his mathematics, like his economics, reveal a tension between method and reality.

The reference to the Taylor series again shows how before "burning the maths", as Marshall said one should, mathematics would also drive much of the research. Here there is an interesting parallel with Newton, who believed that algebra was useful to help us reach a conclusion, but should afterwards be discarded. In both cases, of Newton and Marshall, there is a tension between an underlying philosophical vision, and method. This tension led Newton to continuously revise his methods, which were initially much inspired in Descartes, and Marshall to continuously revise the second volume of the *Principles*, which remained unpublished. The attempt to find certainty in an uncertain reality can well be the source of this tension.

Keynes (1936) later criticised the Marshallian framework of the *Principles*, as developed by Arthur Cecil Pigou (1920), initiating the Cambridge Keynesian tradition (on which see Harcourt, 2003, or Pasinetti, 2005). In fact, after Keynes' (1936) contribution, the Cambridge economic tradition became divided between what may be termed as the Cambridge 'welfare' tradition, which followed Marshall and Pigou, and the Cambridge Keynesian tradition, which rejected the neoclassical framework of Marshall and Pigou. But Keynes' overall conception contrasts not only with the Marshallian neoclassical framework, but also with Newton's search for certainty.

In fact, Keynes' tries to transcend the tension generated by the search for certainty by resorting to an original conception of probability – see Lawson (1985b), Carabelli (1985) and Runde (1994, 2003). Keynes (1921) argues that even when we cannot establish an exact law that relates two propositions with certainty, there is still a logical relation between the two propositions, namely the probability relation, wherein a given conclusion is always related to a given premise with a given probability.

But this does not mean that this probability can be quantified through a numerical value. The numerical value of this probability can only be known in the case of exclusive, exhaustive and equi-probable alternatives, as Lawson (1985b) and Anna Carabelli (1985) explain. This led Keynes (1921) to replace certainty for uncertainty, and exactness for probability, leading to a radically new conception –

see Lawson (1985a). Carabelli (1985: 167) writes that "Keynes seemed to see his work on probability as a sort of anti-*Discours de la méthode*, based on probability, ordinary discourse and common sense rather than on certainty and on analytical reason". Thus, Keynes's conception was a definitive rejection of the Cartesian project.

It is important to understand the differences between Keynes' conception of probability, and the mathematical approach to probability of authors like Bernoulli, Pascal and Huyghens. As Carabelli (1985: 159) explains, Keynes was very unhappy with the conception of probability of authors like Bernoulli, Pascal and Huyghens, who assumed the principle of equal probabilities when there was no sufficient reason to think otherwise. This assumption was fundamental to enable the mathematical treatment of probability, but led one to assume a numerical value without any basis for such, an assumption which Keynes found unsatisfactory. Newton was also against this mathematical approach to probability which characterises the work of Bernoulli, Pascal and Huyghens, due to the lack of a basis for such a mathematical treatment. As Guicciardini (2006: 1736) explains, this led Newton to criticise the mathematical theory of probability:

> "… by the help of philosophical geometers and geometrical philosophers, instead of the conjectures and probabilities that are being blazoned about everywhere, we shall finally achieve a science of nature supported by the highest evidence." (Newton translated and cited in Shapiro 1993, 25)

So like Keynes, Newton was very hostile to the mathematical treatment of probability of his time, which can be found in the contributions of Bernoulli, Pascal and Huyghens. Keynes treatment of probabilities, and his rejection of such mathematical treatment of probability, resolves the tension created by the search for mathematical certainty, avoiding a "condition of strain" that characterised the Cambridge mathematical tradition inspired by Newton, and subsequently the Cambridge economic tradition inspired by Marshall.

Keynes achieves this through a conception where probability is grounded on ordinary language and common sense, and not on Cartesian certainty. Keynes' idea of grounding probability on ordinary language was in fact similar to Newton's approach to mathematics in many ways. As Guicciardini (2006) notes, Newton was satisfied only when his conclusions could be supported by geometrical demonstrations which were "worthy of public utterance". This is, indeed, one of the reasons why Newton wants to keep geometry separated from arithmetic, while using geometry for his proofs: the ability to communicate mathematical results using geometrical methods that can be visualised by everyone without resorting to symbols which may not be clearly understood. For example, concerning the squaring of the curves (what today we call integration, following Leibniz), Newton writes:

> "After the area of some curve has thus been found, careful considerations should be given to fabricating a demonstration of the construction which as far as permissible has no algebraic calculation, so that the theorem embellished with it may turn out worthy of public utterance." (Newton, as cited in Guicciardini 2006: 1734-1735).

This neglect of algebraic calculation after using it can be said to be the Newtonian equivalent of Marshall's "burning of the maths" after using it to reach his results. And like Newton, Marshall also wanted to present his economic theory in a language that was "worthy of public utterance", and thus mathematics is left to the appendix of Marshall's *Principles*. The use of ordinary language that is "worthy of public utterance" continued to play a central role in the Cambridge tradition in the twentieth century, in authors like Moore, Keynes and Wittgenstein, who emphasised the importance of common sense – see John Coates (1996) for a discussion.

The Babylonian Method

As Gay Meeks (2003: 26-28) explains, Keynes was influenced by Hume on this issue. In the Scottish Enlightenment, in which Hume and Smith were central figures, common sense played indeed an important role, as Flavio Comim (2002) explains. Furthermore, unlike the Continental Enlightenment (much influenced by Cartesian pure mathematics), the Scottish Enlightenment interpreted Newton focusing more on his realist approach where common sense played a central role, as Comim (2006) and Leonidas Montes (2006) argue. And thus Keynes' interpretation of Newton can also be seen in light of his reading of Hume, which shows why it is only natural that Keynes was concerned with grounding his work on probability in ordinary language, just like Newton also was concerned with geometrical demonstrations "worthy of public utterance".

This approach ultimately led Keynes to a realist approach to economics, in which he criticised the use of mathematical methods in economics. Keynes argued that economic phenomena are not homogeneous through time, and thus mathematical methods which presuppose the contrary are inappropriate – see Lawson (2003). If we take a Newtonian approach to mathematics to mean a concern with the nature of reality, Keynes can be said to maintain what was termed above as a Newtonian approach to mathematics, in the sense that, for Keynes, mathematics must conform to the analysed reality, in opposition to a Cartesian approach to mathematics where reality (and empirical data) are neglected, which characterises mainstream economics.

In fact, if the Newtonian tradition was already fading away in Marshall's time, vestiges of it remained not only in Marshall's time, but even beyond. After Marshall, we still find Keynes (1936: 297-298) showing a dislike for symbolic mathematics (or as he called it, "symbolic pseudo-mathematical methods") too. Keynes provides the following critique of the use of symbolic mathematics:

> "The object of our analysis is, not to provide a machine, or method of blind manipulation, which will furnish an infallible answer, but to provide ourselves with an organised and orderly method of thinking out particular problems; and, after we have reached a provisional conclusion by isolating the complicating factors one by one, we then have to go back on ourselves and allow, as well as we can, for the probable interactions of the factors amongst themselves. This is the nature of economic thinking. Any other way of applying our formal principles of thought (without which, however, we shall be lost in the wood) will lead us into error. It is a great fault of symbolic pseudo-mathematical methods of formalising a system of economic analysis … that they expressly assume strict independence between the factors involved and lose all their cogency and authority if this hypothesis is disallowed; whereas, in ordinary discourse, where we are not blindly manipulating but know all the time what we are doing and what the words mean, we can keep "at the back of our heads" the necessary reserves and qualifications and the adjustments which we shall have to make later on, in a way in which we cannot keep complicated partial differentials "at the back" of several pages of algebra which assume that they all vanish. Too large a proportion of recent "mathematical" economics are mere concoctions, as imprecise as the initial assumptions they rest on, which allow the author to lose sight of the complexities and

> interdependencies of the real world in a maze of pretentious and unhelpful symbols." (Keynes 1936: 297-298)

Thus, Keynes is explicitly aware of the fact that reality is a interconnected system, and that symbolic mathematics often leads us to use symbols which presuppose that the various components of reality are independent (a closed system characterised by constant conjunctions of the form "if event X then event Y", as Lawson, 2003, notes).

It is not only in Marshall and Keynes, but also in Sraffa, that we find a conception of mathematics in which we must not "lose sight of the complexities and interdependencies of the real world in a maze of pretentious and unhelpful symbols". Sraffa's (1960: vii) insistence on making his argument "easy to follow for the non-mathematical reader", while ignoring the "expert advice" which recommended the use of a different notation (surely one conforming to the symbolic Cartesian approach to mathematics that became dominant), is a consequence of the adoption of a different approach to mathematics by Sraffa too.

Furthermore, even amongst the mathematician who helped Sraffa the most, we find a dislike for purely formal algebra. Amongst the mathematicians mentioned by Sraffa (A. S. Besicovitch, F. P. Ramsey, and Alister Watson), it was Besicovitch who helped Sraffa the most in the writing of his book, as Sraffa (1960: vi) acknowledges. And in the following letter of Besicovitch to Sraffa dated from October 1, 1957, he writes:

> "Dear Sraffa, I have tried to do your problems, but I found myself quite incapable. It is not your fault – you set up the problem quite clearly, but it is just that I could not make myself think on this kind of stuff. *I am at my worst on purely formal algebra*. The problem may by quite easy." (Besicovitch, as cited in Kurz and Salvadori 2007: 188-189, my emphasis-NM)

Thus, the mathematician who helped Sraffa the most is surely not in the formalistic algebraic tradition, and indeed notes that he is at its 'worst' on 'purely formal algebra.' Besicovitch was a student of Andrei A. Markov, who engaged in a separation between geometry and arithmetic, which was to be stressed even more by his son, Andrei A. Markov Jr., who originated the Russian school of constructivist mathematics. We have no evidence, however, at least so far, of whether Besicovitch discussed these issues with Sraffa, or of whether Markov (the father) discussed these issues with Besicovitch.

Furthermore, even Andrei A. Markov Jr. adopted an explicit contructivist approach only later in life (although claiming that he had such an approach in mind for a long time). Wittgenstein, who discussed frequently with Sraffa, was also critical of set theory. Wittgenstein saw both geometry and arithmetic as two different processes of construction according to a rule, where mathematical entities are not simply posited as in set theory, as noted above. This topic could easily have emerged in the discussions between Sraffa and Wittgenstein, but we can never be sure of this, see Davis (1988, 2002) and Sen (2003).

In fact, more convincing evidence of Sraffa's approach to mathematics is provided by Sraffa's (1960) own usage of mathematics. Sraffa's own proofs of the existence of the standard system, the standard commodity, and the reduction to dated quantities of labour, presuppose an algorithm where we find a construction, rather than an appeal to theorems that rely on continuity of a "real line" (such as fixed point theorems) which characterise the Cartesian approach to mathematics.

Many mainstream authors (for example Paul Samuelson) believe that Sraffa's analysis presupposes constant returns to scale, since otherwise the construction of his results consists only in the indication of a rule or process of approximation, rather than in an exact solution that can be obtained through algebraic manipulation. But if Sraffa did adopt a perspective where geometry and arithmetic are kept separated, even the construction of real numbers can only be a rule or process of approximation, and the indication of a rule or process of approximation is all that is needed from a theoretical standpoint. Going further would require empirical analysis, since pure mathematics is not sufficient.

Thus, there is no need to assume constant returns to scale when developing the standard system, the standard commodity, or when approaching the reduction to dated quantities of labour. We do not need an exact proof. Rather, an indication of the rule of construction is all that is needed (as Wittgenstein would say), especially because only empirical reality can tell us the nature of the returns to scale. And thus we can see why it is not only the classical economic theory developed by Sraffa, but also the mathematical methods employed by Sraffa, that do not necessitate the assumption of constant returns to scale.

Vela Velupillai (2008) argues precisely that Sraffa had a constructivist approach to mathematics, although Velupillai focuses more on Errett Bishop's approach (who was more concerned with showing how the Cartesian approach could be expressed in constructivist terms) than on Brouwer's approach (which influenced Wittgenstein) or on the Russian constructivist school of Markov (the son). The Russian constructivist school developed an algorithmic conception while stressing more the differences to what is termed here as the Cartesian approach.

In this constructivist literature, the Cartesian approach to mathematics, developed to its current stage by Cantor, is often termed "classical" mathematics. Of course, the term "classical" mathematics is misleading in this context, since the label "classical" would be more appropriate for the ancient approach that Newton was trying to recover (curiously, just like the term "neoclassical" economics would be more appropriate for the old theory of value that Sraffa was trying to recover). But this ancient approach that Newton develops does not go back only to classical Greece, but beyond, to the Babylonian and Sumerian mathematicians. Indeed Keynes, who engaged in a study of Newton's works (many of which he bought), famously concluded that:

> "Newton was not the first of the age of reason. He was the last of the magicians, the last of the Babylonians and Sumerians" (Keynes 1963: 310)

If the "age of reason" means an age concerned only with Cartesian pure thought, then Newton was certainly an author who had a broader conception of reality. Based on Feynman's (1965) notion of "Babylonian mathematics", and on Keynes' (1963) writings on Newton, Sheila Dow (1990) uses the term "Babylonian method" to designate the methodology of the Keynesian tradition, arguing that this method, charaterised by the use of multiple strands of argument with different starting points, was also used by classical political economists from Smith to Marx.

Dow (1990) argues that mainstream economics, in contrast, is characterised by Cartesian deductivist methods, which are only a particular case of the Babylonian methodology. This is in fact the key difference between the approach to mathematics that underpins mainstream economics, and the one that underpins other approaches from classical political economy to the Cambridge economic tradition.

The point to note here is not whether any modern mathematician or economist was ever able to return to the ancient separation between geometry and arithmetic that Newton was trying to recover. Arguably even Newton himself was still influenced by the Cartesian mixture between geometry and algebra too. And so were the Cambridge economists, who certainly did not dedicate as much time to think about mathematics as Newton did.

Thus, as noted above, Marshall often used Leibnizian notation in the mathematical appendix of the *Principles*, Keynes also uses Leibnizian notation when defining the marginal propensity to consume, and Sraffa indeed uses Cartesian graphs ocasionally not just to represent his results (as Marshall also does), but also to show that in single product industries, when the rate of profit increases, the rate of fall of prices cannot exceed the rate of fall of wages (Sraffa also uses Cartesian graphs to prove the opposite in multiple product industries).

The point to note is that there is a distinction between those that fully embraced the Cartesian approach (where mathematics becomes self-sufficient with its own entities and realities generated by the mixture between geometry and arithmetic), and those for whom mathematics is only a method, or a rule, that generates results to be applied to an external reality. With all their inconsistencies, the Cambridge economists stood on the realist (Newtonian) side, while mainstream economic theory became dominated by a Cartesian approach where mathematics is detached from reality, for the reasons to be further elaborated in the next section.

The Mathematisation of Economics

In the twentieth century, Keynes and Jan Tinbergen engaged in a debate concerning the merits of econometrics, which was then an emerging field. This was a debate in which Keynes criticised the use of econometrics and the mathematisation of the discipline, noting how symbolic mathematics leads us to treat a complex world as if it were composed of independent parts – see Lawson (1985b, 2003).

It is also in this period that game theory emerged, first with the contribution of John von Neumann and Oskar Morgenstern (1944), and soon after with the contribution of John Nash (1951). This is also the moment where Milton Friedman (1953) argued that realism is not necessary for economic theory. And it is at this time that Kenneth Arrow and Gérard Debreu (1954) developed Walrasian general equilibrium analysis, an endeavour to which Lionel McKenzie will also contribute much.

The approach to mathematics that became dominant in this process was the Cartesian approach, where the mixture between continuous geometrical lines and algebraic coordinates can be readily seen in the assumption of continuity used in Arrow and Debreu's (1954) proof of existence of a general equilibrium, which also resorts to arbitrarily small variables akin to Leibniz's infinitesimals, that are simply taken as existent entities.

Another example is Nash's use of the fixed point theorem (which apparently was an obvious procedure to Von Neumann) which again relies on continuity of a function, and the assumption of the existence of equilibrium at a given point which is not constructed. But in this Cartesian approach the existence of mathematical entities is simply assumed, without a contrast of the methods used with an external reality. Rather, mathematics is concerned only with its own realities, following the tendency initiated with set theory, that can be seen in turn as a natural consequence of the mixture between geometry and arithmetic that emerged with Descartes.

In this Cartesian approach, mathematics becomes divorced from reality, and concerned only with possible realities that can be addressed in a formalistic and deductivist way, as it was the case with the "Bourbaki" school that became a very influential school of mathematics in the twentieth century. As Lawson (2003) explains, it is the later approach to mathematics that characterises mainstream economics. In this approach, mathematics is used in a deductivist way, for it attempts to reduce economics to a formalistic approach that presupposes closed system regularities. If the influence of mathematicians like Von Neumann or Nash was pointing in this direction, the contribution of Debreu, and his formalisation of general equilibrium theory, is one of the clearest examples of this approach. Thus Lawson writes:

> "Although Debreu's *Theory of Value* was produced after his move to the US Cowles Commission in the 1950s, Debreu was very much a product of the French Bourbaki 'school' (a group of French mathematicians who argued that mathematical systems should be studied as pure structures devoid of any possible interpretations). It was at the Ecole Normale Supérieure in the 1940s that Debreu came into contact with the Bourbaki teaching. And once trained in this maths, but with his interests aroused by economics, Debreu sought a suitable location to pursue an interest in reformulating economics in terms of this mathematics. It is perhaps not insignificant that his move to the Cowles Commission coincided with the latter's effective acceptance of Bourbakism." (Lawson 2003: 273)

And even when attempts are made to apply mathematics to reality within mainstream economics, as in econometrics, we find a wide gap between econometric theory and econometric practice, as Edward Leamer (1983: 37) points out, where "hardly anyone takes anyone else's data analysis seriously" – see Lawson (2003: 11) for a discussion.

The consequences that Marx extracted from classical political economy concerning the distribution of the surplus also contributed to the definite abandonment of classical political economy, as a reaction to Marx, as Sraffa believed (on which see Garegnani, 2005). But it also had consequences for the acceptance of the Bourbaki approach to mathematics:

> "In particular the emergence of McCarthyite witch-hunts in the context of the Cold War significantly affected the developments in which we are interested. In this climate, the nature of the output of economics faculties became a particularly sensitive matter. And in such a context, the project of mathematising economics proved to be especially attractive. For it carried scientific pretensions but (especially when carried out in the spirit of the Bourbaki approach) was significantly devoid of any necessary empirical content." (Lawson 2003: 274)

This approach to mathematics divorced from reality contrasts with the approach of Newton, which was very influential in Cambridge at a time when the Cartesian approach (concerned with abstract symbols, rather than concrete reality) was increasingly dominant in the European continent.

Given the increasing importance attributed to mathematical-deductivist methods in line with the Cartesian approach, Lawson (2003) argues that mainstream economics is indeed now best characterised by the uncritical acceptance of mathematical-deductivist methods. It is important to note that Lawson is not criticising the use of mathematics in general, but only a particular type of use, namely the Cartesian use of mathematics, disconnected from a concern with reality. Thus, Lawson writes:

> "It is not, and has never been, my intention to oppose the use of formalistic methods in themselves. My primary opposition, rather, is to the manner in which they are everywhere imposed, to the insistence on their being almost universally wielded, irrespective of, and prior to, considerations of explanatory relevance, and in the face of repeated failures. (Lawson 2003: xix)"

Concluding Remarks

Mainstream economics has its origins in the mathematical approach of Walras (1874) and Jevons (1871), and in the marginalist revolution which, despite the efforts of Menger and Marshall, ultimately led to the abandonment of a realist approach to economics. The crucial moment occurred in the twentieth century however, with the econometric revolution, the appearance of game theory, and general equilibrium models. The latter, and especially the work of Debreu (1959), contains the more elaborated version of the Walrasian project, which maintains a Cartesian approach to mathematics, and contrasts with the Newtonian approach to mathematics.

The great success of natural sciences like physics led mainstream economists to believe that mathematico-deductivist methods are essential for economics to become a science. But while natural scientists construct closed systems in laboratory experiences so that physical and chemical phenomena may be exactly measured through mathematico-deductivist methods (or address systems that are relatively closed in our lifespan, such as celestial bodies), mainstream economists posit that human agents behave in a rational, exact and predictable way, so that similar mathematico-deductivist methods can similarly be used in a supposedly “scientific” way, while ignoring the nature of the underlying reality.

Central authors of the Cambridge economic tradition, like Marshall, Keynes, and Sraffa, adopted a different approach to mathematics, which is not deductivist, and existed already in Newton. Marshall’s method, based on physics, still constrained him from developing consistently his insights regarding evolutionary biology, and the correspondent idea that reality is an organic process. Keynes went further in his elaboration of an economic theory where the economy is an organic whole. And Sraffa’s approach certainly takes into account the economy as a whole, while showing the limitations of Marshall’s partial equilibrium analysis.

There were, of course, many differences between the specific economic theories of Marshall, Keynes and Sraffa. But if a coherent framework is to be found in the Cambridge economic tradition, it can be found at the level of ontology, within a conception where reality is an organic *process*. Other characteristics, like a Newtonian approach to mathematics, are a consequence of an attempt to analyse this reality, which also leads to the need for diverse methods, and thus to what Dow calls a “Babylonian” approach. This approach contrasts with mainstream economics, where unity is found at the level of the method, a method which consists in Cartesian deductivist mathematics, as Lawson (2003) and Dow (1990, 2003) argue.

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